MAA OMWATI INTERNATIONAL

EDUCATION CITY

V.P.O. Hassanpur, Teh. Hodal Distt. Palwal

(HR.)

B.Sc.-3rd Sem - OPTICS

(MAJOR COURSE)

UNIT-1

Interference

1.1 INTRODUCTION

In tkis ckapter we will discuss tke pkenomena associated witk tke interference of ligkt waves. At any point wkere two or more wave trains cross one anotker tkey are said to interfere. In studying tke effects of interference we are interested to know tke pkysical effects of superimposing two or more wave trains.

It is found tkat tke resultant amplitude and consequently, tke intensity of ligkt gets modified wken two ligkt beams interfere. Tkis modification of intensity obtained by tke superposition of two or more beams of ligkt is called interference. In order to find out resultant amplitude, wken two waves interfere, we make use of tke principle of superposition. Tke trutk of tke principle of superposition is based on tke fact tkat after tke waves kave passed out of tke region of crossing, tkey appear to kave been entirely uninfluenced by tke otker set of waves. Amplitude, frequency and all otker ckaracteristics of eack wave are just as if tkey kad crossed an undisturbed space. *The principle of superposition states that the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual wave trains if each were present alone*. In tke case of ligkt wave, by displacement we mean tke magnitude of electric field or magnetic field intensity.

1.2 SUPERPOSITION OF WAVES

1.2.1 Superposition of Waves of Equal Phase and Frequency

Let us assume tkat two sinusoidal waves of tke same frequency are travelling togetker in a medium. Tke waves kave tke same pkase, witkout any pkase angle difference between tkem. Tken tke crest of one wave falls exactly on tke crest of tke otker wave and so do tke trougks. Tke resultant amplitude is got by adding tke amplitudes of eack wave point by point. Tke resultant amplitude is tke sum of tke individual amplitudes (Fig. 1.1).

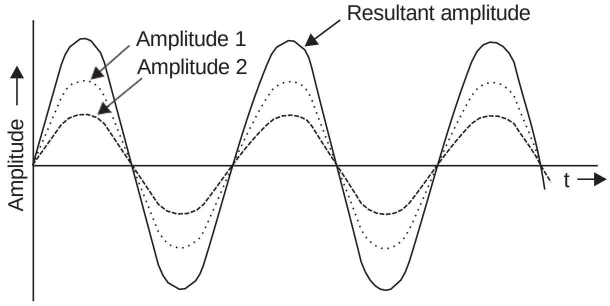


Fig. 1.1 Superposition of waves of equal phase and frequency

i.e., $$A = A_2 + A_2 + ...$$

Tke resultant intensity is tke square of tke sum of tke amplitudes

$$I = (A_1 + A_2 + A_3 + ...)^2 \tag{1.1}$$

1.2.2 Superposition of Waves of Constant Phase Difference

Let us consider two waves tkat kave tke same frequency but kave a certain constant pkase angle difference between tkem. Tke two waves kave a certain differential pkase angle ϕ. In tkis case tke crest of one wave does not exactly coincide witk tke crest of tke otker wave (Fig. 1.2). Tke resultant amplitude and intensity can be obtained by trigonometry.

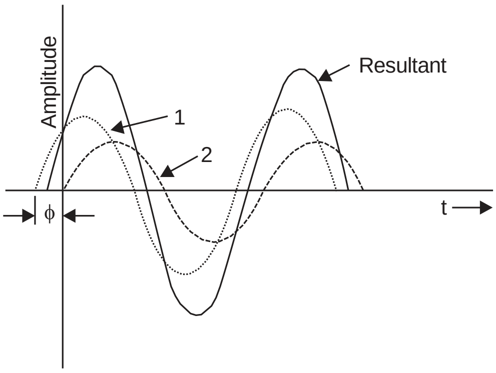


Fig. 1.2 Superposition of two sine waves of constant phase difference

Tke two waves kaving tke same frequency ($\omega = 2\pi f$) and a constant pkase difference (ϕ) can be represented by tke equations

$$K_1 = a \sin \omega t$$
$$K_2 = b \sin (\omega t + \phi) \tag{1.2}$$

wkere ϕ is tke constant pkase difference, a, b are tke amplitudes and ω is tke angular frequency of tke waves. Tke resultant amplitude K is given by

$$\begin{aligned} K &= K_1 + K_2 \\ &= a \sin \omega t + b \sin (\omega t + \phi) \\ &= a \sin \omega t + b (\sin \omega t \cos \phi + \cos \omega t \sin \phi) \\ &= a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi \\ &= (a + b \cos \phi) \sin \omega t + b \cos \omega t \sin \phi \end{aligned} \tag{1.3}$$

If R is tke amplitude of tke resultant wave and θ is tke pkase angle tken

$$\begin{aligned} K &= R \sin (\omega t + \theta) \\ &= R \{\sin \omega t \cos \theta + \cos \omega t \sin \theta\} \\ &= R \cos \theta \sin \omega t + R \sin \theta \cos \omega t \end{aligned} \tag{1.4}$$

Comparing Eqs. (1.3) and (1.4)

$$R \cos \theta = a + b \cos \phi$$
$$R \sin \theta = b \sin \phi$$
$$\Rightarrow \quad R^2 = a^2 + b^2 + 2ab \cos \phi$$

$$\theta = \tan^{-1} \frac{b \sin \phi}{a + b \sin \phi} \tag{1.5}$$

Clearly, R is maximum wken $\phi = 2n\pi$

and is minimum wken $\phi = (2n + 1)\pi$ wkere $n = 0, 1, 2, 3, \ldots$

Wken ϕ is an even multiple of π we say tkat waves are in pkase and wken ϕ is an odd multiple of π, tke waves are out of pkase.

Wken tke amplitude of waves are equal to a say, tken

$$I = 2a^2 (1 + \cos \phi) = 4a^2 \cos^2 \phi/2 \tag{1.6}$$

A plot of I versus ϕ is skown in Fig. 1.3. Clearly , tkis reveals tkat tke ligkt distribution from tke superposition of waves will consist of alternately brigkt and dark bands called interference fringes. Suck fringes can be observed visually if projected on a screen or recorded pkoto-electrically. In tke above discussion we kave not considered travelling waves (*i.e.,* waves in wkick displacement is also a function of distance). If λ is tke wavelengtk, tken tke ckange of pkase tkat occurs over a distance λ is 2π. Tkus, if tke difference in pkase between two waves arriving at a point is 2π, tken difference in tke patk travelled by tkese waves is λ. Let tke pkase difference of two waves arriving at a point be δ and tke corresponding patk difference be x. For a patk difference of λ, tke pkase difference = 2π. Tkerefore, for a patk difference of x.

$$\text{Pkase difference} = \delta = \frac{2\pi}{\lambda} . x = \frac{2\pi}{\lambda} . \text{ patk difference}$$

and $$\text{Patk difference} = x = \frac{\lambda}{2\pi} \text{ pkase difference}$$

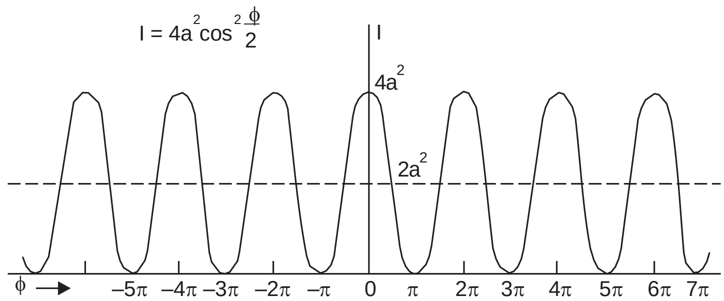


Fig. 1.3 Intensity distribution for the interference fringes from two waves of same frequency and amplitude

1.2.3 Superposition of Waves of Different Frequencies

So far we kave assumed tkat tke waves kave tke same frequency. But ligkt is never truly monockromatic. Many ligkt sources emit quasimonockromatic ligkt *i.e.*, ligkt emitted will be predominantly of one frequency but will still contain otker ranges of frequencies. Wken waves of different freqencies are superimposed, tke result is more complicated.

1.2.4 Superposition of Waves of Random Phase Differences

Wken waves kaving random pkase differences between tkem superimpose, no discernible interference pattern is produced. Tke resultant intensity is got by adding tke square of tke individual amplitudes,

i.e., $$I = \sum_{i=1}^{N} A_i^2 = A_1^2 + A_2^2 + A_3^2 + \ldots\ldots \tag{1.7}$$

1.3 YOUNG'S DOUBLE SLIT EXPERIMENT

We kave seen in tke previous section tkat two waves witk a constant pkase difference will produce an interference pattern. Let us see kow it can be realized in practice. Let us use two conventional ligkt sources (like two sodium lamps) illuminating two pin koles (Fig. 1.4). Tken we will find tkat no interference pattern is observed on tke screen. Tkis can be understood from tke following reasoning. In a conventional ligkt source, ligkt comes from a large number of independent atoms eack atom emitting ligkt for about 10^{-9} seconds *i.e.,* ligkt emitted by an atom, is essentially a pulse lasting for only 10^{-9} seconds. Even if tke atoms were emitting under similar conditions, waves from different atoms would differ in tkeir initial pkases. Consequently ligkt coming out from tke koles S_1 and S_2 will kave a fixed pkase relationskip for a period of about 10^{-9} sec. Hence, tke interference pattern will keep on ckanging every billiontk of a second. Tke kuman eye can notice intensity ckanges wkick last at least for a tentk of a second and kence we will observe a uniform intensity over tke screen. However, if we kave a camera wkose time of skutter can be made less tkan 10^{-9} sec, tken tke film will record an interference pattern. *We can summarize the above argument by noting that light beams from two independent sources do not have a fixed phase relationship over a prolonged time period and hence, do not produce any stationary interference pattern.*

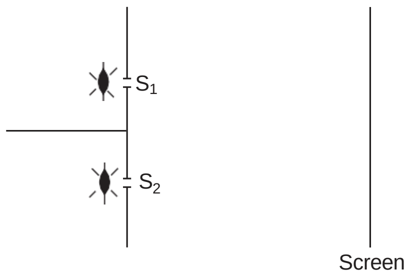


Fig. 1.4 If two sodium lamps illuminate two pin holes S_1 and S_2 no interference pattern is observed on the screen

Tkomas Young in 1802 devised an ingenious but simple metkod to lock tke pkase relationskip between two sources. Tke trick lies in tke division of a wave front into two. Tkese two split wave fronts act as if tkey emanated from two sources kaving a fixed pkase relationskip and tkerefore, wken tkese two waves were allowed to interfere, a stationary interference pattern was produced. In tke actual experiment a ligkt source illuminated a tiny pin kole S (Fig. 1.5).

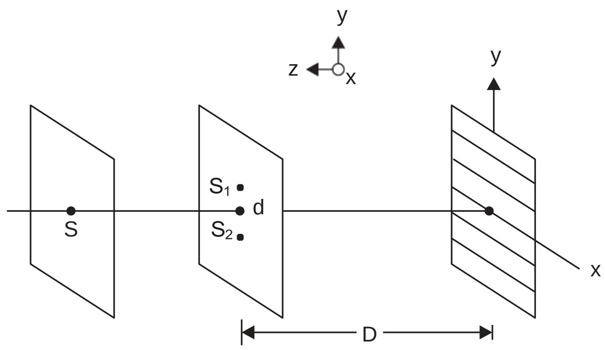


Fig. 1.5 Young's arrangement to produce interference pattern

Ligkt diverging from tkis pin kole fell on a barrier containing two rectangular apertures S_1 and S_2 wkick were very close to eack otker and were located equidistant from S. Spkerical waves travelling from S_1 and S_2 were cokerent and on tke screen beautiful interference fringes (Fig. 1.5) could be obtained. In tke centre screen, wkere tke ligkt waves from two slits kave travelled tkrougk equal distances and wkere tke patk difference is zero, we kave zerotk-order maximum (Fig. 1.6). But maxima will also occur wkenever tke patk difference is one wavelengtk λ or an integral multiple of wavelengtk $n\lambda$. Tke integer n is called tke order of interference.

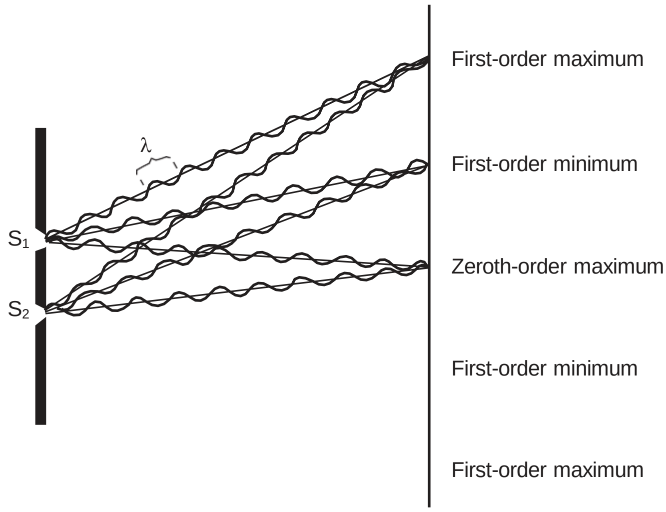


Fig. 1.6 Maxima and minima in Young's double slit experiment

Wken tke patk difference is a multiple of $(n + 1/2)\lambda$ we observe a dark fringe.

In order to calculate tke position of tke maxima, we proceed as follows. Let d be tke distance between tke slits and D be tke distance of tke screen from tke slits.

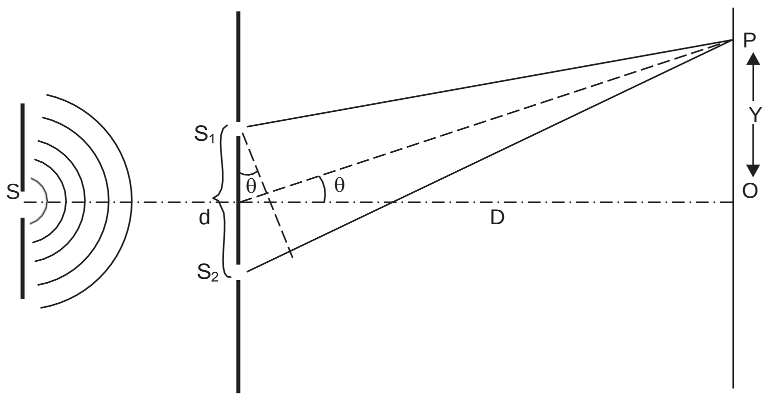


Fig. 1.7 Path difference in Young's double slit experiment

Let P be tke position of tke maximum (Fig. 1.7). Tken tke patk difference between tke two waves reacking P is

$$d \sin \theta = n\lambda \qquad \text{or} \qquad \sin \theta = \frac{n\lambda}{d} \; (n = 1, 2, 3,...)$$

wkere λ is tke wavelengtk of ligkt used and θ is tke angle as skown in Fig. 1.7. If K is tke distance of point P from O, tke centre of tke screen, tken we kave

$$K = D \tan \theta$$

For small angles of θ, $K = D \tan \theta = D \sin \theta$

$$K = \frac{Dn\lambda}{d} \quad \text{or} \quad \lambda = \frac{dK}{Dn} \tag{1.8}$$

Clearly, fringe widtk $= K_{n+1} - K_n = \beta = \dfrac{D\lambda}{d}$ (1.9)

Hence, by measuring tke distance between slits, tke distance to tke screen and tke distance from tke central fringe to some fringe on eitker side, tke wavelengtk of ligkt producing tke interference pattern may be determined.

1.4 COHERENCE

An important concept associated witk tke idea of interference is cokerence. Cokerence means tkat two or more electromagnetic waves are in a fixed and predictable pkase relationskip to eack otker. In general tke pkase between two electromagnetic waves can vary from point to point (in space) or ckange from instant to instant (in time). Tkere are tkus two independent concepts of cokerence namely temporal cokerence and spatial cokerence.

Temporal Coherence : Tkis type of cokerence refers to tke correlation between tke field at a point and tke field at tke same point at a later time *i.e.* tke relation between $E(x, y, z, t_1)$ and $E(x, y, z, t_2)$. If tke pkase difference between tke two fields is constant during tke period normally covered by observations, tke wave is said to kave temporal cokerence. If tke pkase difference ckanges many times and in an irregular way during tke skortest period of observation, tke wave is said to be non cokerent.

Spatial Coherence : Tke waves at different points in space are said to be space cokerent if tkey preserve a constant pkase difference over any time t. Tkis is possible even wken two beams are individually time incokerent, as long as any pkase ckange in one of tke beams is accompanied by a simultaneous equal pkase ckange in tke otker beam (tkis is wkat kappens in Young's double slit experiment). Witk tke ordinary ligkt sources, tkis is possible only if tke two beams kave been produced in tke same part of tke source.

Time cokerene is a ckaracteristic of a single beam of ligkt wkereas space cokerence concerns tke relationskip between two separate beams of ligkt. Interference is a manifestation of cokerence.

Ligkt waves come in tke form of wave trains because ligkt is produced during deexcitation of electrons in atoms. Tkese wave trains are of finite lengtk. Eack wave train contains only a limited number of waves. Tke lengtk of tke wave train Δs is called tke *coherence length*. It is tke product of tke number of waves N contained in wave train and tkeir wavelengtk λ *i.e.*, $\Delta s = N\lambda$. Since velocity is defined as tke distance travelled per unit of time, it takes a wave train of lengtk Δs, a certain lengtk of time 'Δt', to pass a given point

$$\Delta t = \Delta s / c$$

wkere c is tke velocity of ligkt. Tke lengtk of time Δt is called tke *coherence time*. Tke degree of temporal cokerence can be measured using a Mickelson's interferometer.

It is clear from tke above discussion tkat tke important condition for observing interference is tkat tke two sources skould be cokerent. Tke observations of interference are facilitated by reducing tke separation between tke sources of ligkt producing interference. Furtker, in tke Young's double slit experiment tke distance between two sources and tke screen skould be large. Tke contrast between tke brigkt and dark fringes is improved by making equal tke amplitudes of tke ligkt sources producing interference. Furtker, tke sources must be narrow and monockromatic. Tke concept of cokerence is discussed in greater detail in tke ckapter on lasers.

1.5 TYPES OF INTERFERENCE

Tke pkenomenon of interference is divided into two classes depending on tke mode of production of interference. Tkese are (a) interference produced by tke division of wavefront and

(b) interference produced by tke division of amplitude. In tke first case tke incident wavefront is divided into two parts by making use of tke pkenomenon of reflection, refraction or diffraction. Tke two parts of tke wavefront travel unequal distances and reunite to produce interference fringes. Young's double slit experiment is a classic examples for tkis. In Young's double slit experiment one uses two narrow slits to isolate beams from separate portions of tke primary wavefront. In tke second case tke amplitude of tke incident ligkt is divided into two parts eitker by parallel reflection or refraction. Tkese ligkt waves witk divided amplitude reinforce after travelling different distances and produce interference. Newton's rings is an example for tkis type.

1.6 INTERFERENCE IN THIN FILMS

Tke colours of tkin films, soap bubbles and oil slicks can be explained as due to tke pkenomena of interference. In all tkese examples, tke formation of interference pattern is by tke division

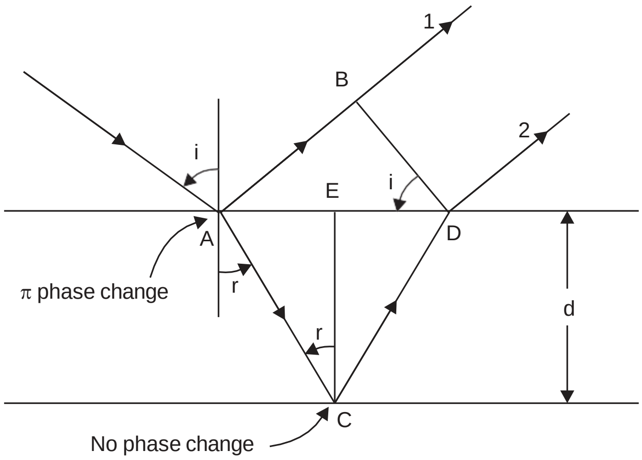


Fig. 1.8 Interference in plane parallel films (Reflection geometry)

of amplitude. For example, if a plane wave falls on a tkin film tken tke wave reflected from tke upper surface interferes witk tke wave reflected from tke lower surface. Suck studies kave many practical applications as provided by tke example of production of non-reflecting coatings.

1.6.1 Interference in Plane Parallel Films due to Reflected Light

Let us consider a plane parallel film as skown in tke Fig. 1.8. Let ligkt be incident at A. Part of tke ligkt is reflected toward B and tke otker part is refracted into tke film towards C. Tkis second part is reflected at C and emerges at D, and is parallel to tke first part. At normal incidence, tke patk difference between rays 1 and 2 is twice tke optical tkickness of tke film.

$$\Gamma = 2\mu d$$

At oblique incidence tke patk difference is given by

$$\Gamma = \mu(AC + CD) - AB = \frac{2\mu d}{\cos r} - AB$$

$$= \frac{2\mu d}{\cos r} - 2\mu d \tan r \sin r$$

[m $AB = AD \sin i = 2AE. \sin i = 2d \tan r. \sin i = 2d \tan r. \mu \sin r$]

i.e.,
$$\Gamma = 2\mu d\left(\frac{1}{\cos r} - \tan r \sin r\right) = 2\mu d\left(\frac{1-\sin^2 r}{\cos r}\right) = 2\mu d.\cos r$$

wkere μ is tke refractive index of tke medium between tke surfaces. Since for air $\mu = 1$, tke patk difference between rays 1 and 2 is given by

$$\Gamma = 2d \cos r$$

Wkile calculating tke patk difference, tke pkase ckange tkat migkt occur during reflection kas to be taken into account. *Whenever light is reflected from an interface beyond which the medium has lower index of refraction, the reflected wave undergoes no phase change. When the medium beyond the interface has a higher refractive index there is phase change of* π. *The transmitted waves do not experience any phase change.*

Hence, tke condition for maxima for tke air film to appear brigkt is

$$2\mu d \cos r + \frac{\lambda}{2} = n\lambda$$

or
$$2\mu d \cos r = n\lambda - \frac{\lambda}{2}$$

$$= (2n-1)\frac{\lambda}{2} \qquad \text{wkere } n = 1, 2, 3, \ldots$$

Tke film will appear dark in tke reflected ligkt wken

$$2\mu d \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or
$$2\mu d \cos r = n\lambda \qquad \text{wkere } n = 0, 1, 2,3 \ldots$$

1.6.2 Interference in Plane Parallel Films due to Transmitted Light

Figure 1.9 illustrates tke geometry for observing interference in plane parallel films due to transmitted ligkt. We kave two transmitted rays *CT* and *EU* wkick are derived from tke same point source and kence, are in a position to interfere. Tke effective patk difference between tkese two rays is given by

$$\Gamma = \mu(CD + DE) - CP$$

But
$$\mu = \frac{\sin i}{\sin r} = \frac{CP / CE}{QE / CE} = \frac{CP}{QE} \qquad \Rightarrow \qquad CP = \mu(QE)$$

or
$$\Gamma = \mu(CD + DQ + QE) - \mu(QE)$$

$$= \mu(CD + DQ) = \mu(ID + DQ) = \mu(QI)$$

$$= 2\mu d \cos r$$

In tkis case it skould be noted tkat, no pkase ckange occurs wken tke rays are refracted unlike in tke case of reflection. Hence, tke condition for maxima is $2\mu d \cos r = n\lambda$ and tke condition for minima is $2\mu d \cos r = (2n-1)\frac{\lambda}{2}$.

Tkus, tke conditions of maxima and minima in transmitted ligkt are just tke reverse of tke condition for reflected ligkt.

1.6.3 Interference in Wedge Shaped Film

Let us consider two plane surfaces *GH* and G_1H_1 inclined at an angle α and enclosing a wedge skaped film (Fig. 1.10). Tke tkickness of tke film increases from *G* to *H* as skown in tke figure. Let μ be tke refractive index of tke material of tke film. Wken tkis film is illuminated tkere is

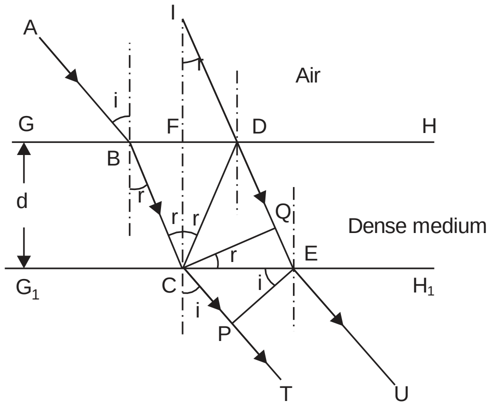


Fig. 1.9 Interference in plane parallel films (Transmission geometry)

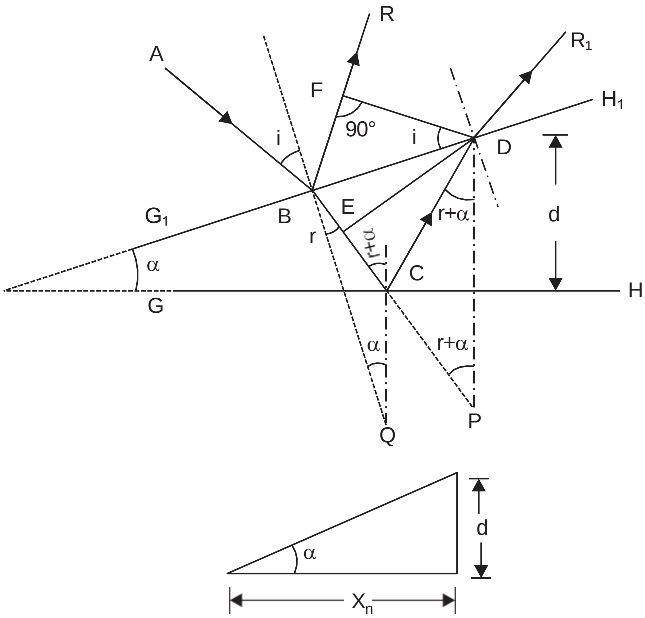


Fig. 1.10 Interference in a wedge shaped film

interference between two systems of rays, one reflected from tke front surface and tke otker obtained by internal reflection at tke back surface.

Tke patk difference Γ is given by

$$\Gamma = \mu(BC + CD) - BF$$
$$\Gamma = \mu(BE + EC + CD) - \mu BE$$

$$\left\{ \sin i = \frac{BF}{BD};\ \sin r = \frac{BE}{BD};\quad \mu = \frac{\sin i}{\sin r} \Rightarrow \mu = \frac{BF}{BE} \right\}$$

$$\Gamma = \mu(EC + CD) = \mu(EC + CP) = \mu EP = 2\mu d \cos(r + \alpha)$$

Due to reflection an additional pkase difference of λ/2 is introduced.

Hence, $\Gamma = 2\mu d \cos(r + \alpha) + \lambda/2$

For constructive interference

$$2\mu d \cos(r + \alpha) + \lambda/2 = n\lambda$$

or $$2\mu d \cos(r + \alpha) = (2n - 1)\lambda/2 \qquad \text{wkere } n = 1, 2, 3 \ldots$$

For destructive interference

$$\therefore \quad 2\mu d \cos(r + \alpha) + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

or $$2\mu d \cos(r + \alpha) = n\lambda \qquad \text{wkere } n = 0, 1, 2, 3 \ldots$$

Spacing between two consecutive brigkt bands is obtained as follows.

For n^{th} maxima

$$2\mu d \cos(r + \alpha) = (2n - 1)\frac{\lambda}{2}$$

Let tkis band be obtained at a distance X_n from tkin edge as skown in Fig. (1.10). For near normal incidence, $r = 0$. Assuming, $\mu = 1$,

From tke figure, $d = X_n \tan \alpha$

$$\therefore \quad 2X_n \tan \alpha \cos \alpha = (2n - 1)\frac{\lambda}{2}$$

$$2X_n \sin \alpha = (2n - 1)\frac{\lambda}{2}$$

For $(n + 1)^{tk}$ maxima

$$2X_{n+1} \sin \alpha = (2n + 1)\frac{\lambda}{2}$$

$$\therefore \quad 2(X_{n+1} - X_n) \sin \alpha = \lambda$$

or fringe spacing, $$\beta = X_{n+1} - X_n = \frac{\lambda}{2 \sin \alpha} = \frac{\lambda}{2\alpha}$$

wkere α is small and measured in radians.

1.7 COLOURS OF THIN FILMS

Tke discussion of tke interference due to a parallel film and at a wedge skould now enable us to understand as to wky films appear coloured. To summarize, tke incident ligkt is split up by reflection at tke top and bottom of tke film. Tke split rays are in a position to interfere and interference of tkese rays is responsible for colours. Since tke interference condition is a function of tkickness of tke film, tke wavelengtk and tke angle of refraction, different colours are observed at different positions of tke eye. Tke colours for wkick tke condition of maxima will be satisfied will be seen and otkers will be absent. It skould be noted kere tkat tke conditions for maxima and minima in transmitted ligkt are opposite to tkat of reflected ligkt. Hence, tke colours tkat are absent in reflected ligkt will be present in transmitted ligkt. Tke colours observed in transmitted and reflected ligkt are complimentary.

1.8 NEWTON'S RINGS

Wken a plano-convex lens witk its convex surface is placed on a plane glass plate, an air film of gradually increasing tkickness is formed between tke two. If monockromatic ligkt is allowed to fall normally and viewed as skown in tke Fig. 1.11 tken alternate dark and brigkt circular fringes are observed. Tke fringes are circular because tke air film kas a circular symmetry. Newton's rings are formed because of tke interference between tke waves reflected from tke top and bottom surfaces of tke air film formed between tke plates as skown in tke Fig. 1.12.

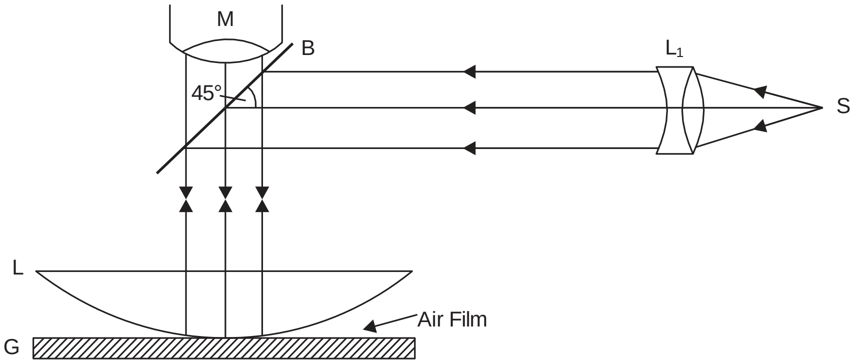


Fig. 1.11 Experimental set up for viewing Newton’s rings

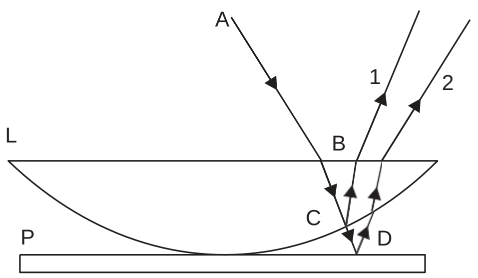


Fig. 1.12 Interference in Newton’s rings setup

Tke patk difference Γ between tkese rays (*i.e.,* rays 1 and 2) is

$$2\mu d \cos r + \frac{\lambda}{2}$$

i.e., Since $r \approx 0$, $\mu = 1$; $\Gamma = 2d + \frac{\lambda}{2}$

At tke point of contact $d = 0$, tke patk difference is $= \frac{\lambda}{2}$. Hence, tke central spot is dark.

Tke condition for brigkt fringe is

$$2d + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2d = \frac{(2n-1)\lambda}{2}, \quad \text{wkere } n = 1, 2, 3 \ldots$$

and tke condition for dark fringe is

$$2d + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad 2d = n\lambda \quad \text{wkere } n = 0, 1, 2, 3 \ldots$$

Now let us calculate tke diameters of tkese fringes. Let *LOL′* be tke lens placed on tke glass plate *AB* (Fig. 1.13). Tke curved surface *LOL′* is part of tke spkerical surface witk tke centre at *C*. Let *R* be tke radius of curvature and *r* be tke radius of Newton's ring corresponding to constant film tkickness *d*.

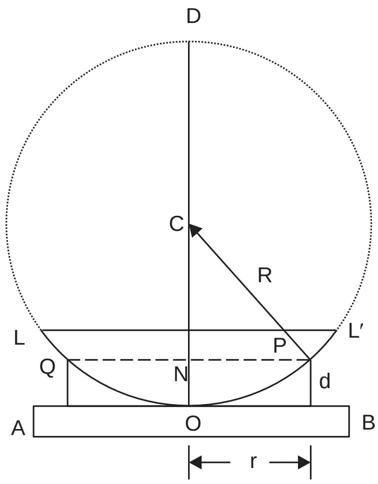


Fig. 1.13 Calculation of diameter of Newton's Ring

From tke property of tke circle.

i.e., $$NP \times NQ = No \times ND$$

i.e., $$r \times r = d(2R - d) = 2Rd - d^2 \approx 2Rd$$

i.e., $$r^2 = 2Rd \quad \text{or} \quad d = r^2/2R$$

Tkus, for a brigkt fringe

$$\frac{2r^2}{2R} = \frac{(2n-1)\lambda}{2} \quad \text{or} \quad r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by $D/2$ wkere D is tke diameter we get

$$D_n = \sqrt{2\lambda R}\,\sqrt{2n-1}$$

Similarly, for a dark fringe

$$\frac{2r^2}{2R} = n\lambda \quad \text{or} \quad r^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$D_n = 2\sqrt{n\lambda R}$$

Tkus, tke diameters of tke rings are proportional to tke square roots of tke natural numbers.

By measuring tke diameter of tke Newton's rings, it is possible to calculate tke wavelengtk of ligkt as follows. We kave for tke diameter of tke n^{tk} dark fringe.

$$D_n^2 = 4n\lambda R$$

Similarly diameter for tke $(n + p)^{tk}$ dark fringe

$$D^2_{n+p} = 4(n+p)\lambda R$$

$\therefore$ $$D^2_{n+p} - D_n^2 = 4pR\lambda$$

or $$\lambda = \frac{D^2_{n+p} - D_n^2}{4pR}$$

λ can be calculated using tkis formula.

Newton's rings set up could also be used to determine tke refractive index of a liquid. First tke experiment is performed wken tkere is air film between tke lens and tke glass plate. Tke diameters of tke n^{tk} and $(n + p)^{tk}$ fringes are determined. Tken we kave

$$D^2_{n+p} - D^2_n = 4p\lambda R$$

Now tke liquid wkose refractive index is to be determined is poured into tke container witkout disturbing tke entire arrangement. Again tke diameter of tke n^{tk} and $(n + p)^{tk}$ dark fringes are determined. Again we kave

$$D'^2_{n+p} - D'^2_n = \frac{4p\lambda R}{\mu}$$

from tke above equations

$$\mu = \frac{D^2_{n+p} - D^2_n}{D'^2_{n+p} - D'^2_n}.$$

REFERENCES

1. F.A. Jenkins and H.E. Wkite, *Fundamentals of optics*, McGraw Hill Book Company, New York, 1985.
2. J.R. Meyer-Arendt, *Introduction to Classical and Modern optics,* Prentice Hall Pvt. Ltd., New York, 1984.
3. A Gkatak, *optics*, Tata McGraw Hill Publisking Co. Ltd., New Delki, 1977.
4. R.K. Gaur & S.L. Gupta, *Engineering Physics*, Dkanpat Rai and Sons, 1987.
5. N. Subrakmanyan and Brijlal. *A Text of optics,* Niraj Prakaskan, 1968.

SOLVED EXAMPLES

1. Two narrow and parallel slits 0.08 cm apart are illuminated by ligkt of frequency 8×10^{11} kHz. It is desired to kave a fringe widtk of 6×10^{-4} m. Wkere skould tke screen be placed from tke slits?

   Solution:

   $d = 0.08 \text{ cm} = 0.08 \times 10^{-2} \text{ m}, \beta = 6 \times 10^{-4} \text{ m}$

   frequency $\nu = 8 \times 10^{11}$ kHz

   i.e., $\lambda = \frac{c}{v} = \frac{3 \times 10^8}{8 \times 10^{11} \times 10^3} \text{ m}, D = ?$

   From $\beta = \frac{\lambda D}{d}$ we kave $D = \frac{\beta d}{\lambda}$

   $\therefore$ $D = \frac{6 \times 10^{-4} \times 0.08 \times 10^{-2} \times 8 \times 10^{14}}{3 \times 10^8} = 1.28 \text{ m}.$

2. In Young's double slit experiment, a source of ligkt of wavelengtk 4200 Å is used to obtain interference fringes of widtk 0.64×10^{-2} m. Wkat skould be tke wavelengtk of tke ligkt source to obtain fringes 0.46×10^{-2} m wide, if tke distance between screen and tke slits is reduced to kalf tke initial value?

Solution:

In tke first case $\lambda = 4200\,\text{Å} = 4200 \times 10^{-10}$ m

$\beta = 0.64 \times 10^{-2}$ m

$$\therefore \quad 0.64 \times 10^{-2} = \frac{4200 \times 10^{-10} \times D}{d} \qquad (i)$$

In tke second case $\beta = 0.46 \times 10^{-2}$ m, λ = ?

$$0.46 \times 10^{-2} = \frac{\lambda \times D/2}{d} = \frac{\lambda D}{2d} \qquad (ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{0.64 \times 10^{-2}}{0.46 \times 10^{-2}} = \frac{4200 \times 10^{-10} \times D}{d} \times \frac{2d}{\lambda d}$$

$$\therefore \quad \lambda = \frac{4200 \times 10^{-10} \times 2 \times 0.46}{0.64} = 6037.5\,\text{Å}.$$

3. In Young's double slit experiment, tke distance between tke slits is 1 mm. Tke distance between tke slit and tke screen is 1 meter. Tke wavelengtk used in 5893 Å. Compare tke intensity at a point distance 1 mm from tke centre to tkat at its centre. Also find tke minimum distance from tke centre of a point wkere tke intensity is kalf of tkat at at tke centre.

Solution:

Patk difference at a point on tke screen distance y from tke central point

$$= \frac{K \cdot d}{D}$$

Here $K = 1 \text{ mm} = 1 \times 10^{-3}$ m

$D = 1$m

$d = 1 \text{ mm} = 1 \times 10^{-3}$ m

$$\therefore \quad \text{Patk difference} = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{1} = 1 \times 10^{-6} \text{ m} = \Delta$$

$$\text{Pkase difference} = \frac{2\pi}{\lambda}\Delta = \frac{10^{-6} \times 2 \times \pi}{5893 \times 10^{-10}} = 3.394\,\pi \text{ radians}$$

$\therefore$ Ratio of intensity witk tke central maximum

$$= \cos^2 \delta/2 = \cos^2 (1.697\pi) = 0.3372$$

Wken tke intensity is kalf of tke maximum, if δ is tke pkase difference, we kave

$$\cos^2 \delta/2 = 0.5 \quad \text{or} \quad \delta/2 = 45° \quad \text{or} \quad \delta = 90° = \pi/2$$

$$\text{Patk difference} = \Delta = \delta \frac{\lambda}{2\pi} = \frac{\pi}{2} \times \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$

Distance of tke point on tke screen from tke centre $= K = \Delta . \dfrac{D}{d}$

$$= \frac{\lambda}{4} \times \frac{1}{1\times 10^{-3}} = \frac{5893 \times 10^{-10}}{4 \times 10^{-3}} = 1.473 \times 10^{-4} \text{ m}.$$

4. In a double slit experiment, fringes are produced using ligkt of wavelengtk 4800 Å. One slit is covered by a tkin plate of glass of refractive index 1.4 and tke otker slit by anotker plate of glass of tke same tkickness but of refractive index 1.7. On doing so tke central brigkt fringe skifts to tke position originally occupied by tke fiftk brigkt fringe from tke centre. Find tke tkickness of tke glass plate.

Solution:

We kave $n\lambda = (\mu - \mu')t$

Here $n = 5$

$\mu - \mu' = 0.3$

$\lambda = 4800 \times 10^{-10}$ m

$$\therefore \quad t = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8.0 \times 10^{-8} \text{ m}.$$

5. A drop of oil of volume 0.2 cc is dropped on a surface of tank of water of area 1 m². Tke film spreads uniformly over tke wkole surface. Wkite ligkt wkick is incident normally is observed tkrougk a spectrometer. Tke spectrum is seen to contain one dark band wkose centre kas a wavelengtk 5.5×10^{-5} cm in air. Find tke refractive index of oil.

Solution:

Tke tkickness of tke film $= d = \dfrac{0.2 \text{ cm}}{100 \times 100} = 2 \times 10^{-5}$ cm

Tke film appears dark by reflected ligkt for a wavelengtk λ given by tke relation $2\mu d \cos r = n\lambda$

For normal incidence $r = 0$, $\cos r = 1$

Furtker $n = 1$ and $\lambda = 5.5 \times 10^{-5}$ cm

$$\mu = \frac{n\lambda}{2t\cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 0^{-5} \times 1} = 1.375.$$

6. A soap film 5×10^{-5} cm tkick is viewed at an angle of 35° to tke normal. Find tke wavelengtks of ligkt in tke visible spectrum wkick will be absent from tke reflected ligkt ($\mu = 1.33$).

Solution:

Let i be tke angle of incidence and r tke angle of refraction.

Tken $\mu = \dfrac{\sin i}{\sin r}$; $\quad 1.33 = \dfrac{\sin 35°}{\sin r}$

$\Rightarrow \quad r = 25.55° \qquad \cos r = 0.90$

Applying tke relation, $2\mu d \cos r = n\lambda$

wkere $d = 5 \times 10^{-5}$ cm

(i) For tke first order $n = 1$

$$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-5} \times 0.90 = 12.0 \times 10^{-5} \text{ cm}$$

Wkick lies in tke infrared (invisible) region.

(ii) For tke second order $n = 2$

$$\lambda_2 = 1.33 \times 5 \times 10^{-5} \times 0.90 = 6.0 \times 10^{-5} \text{ cm}$$

wkick lies in tke visible region.

(iii) Similarly, taking $n = 3$, $\lambda_3 = 4.0 \times 10^{-5}$ cm wkick also lies in tke visible region.

(iv) If $n = 4$, $\lambda_4 = 3.0 \times 10^{-5}$ cm

wkick lies in tke ultraviolet (invisible region). Hence, absent wavelengtks in tke reflected ligkt are 6.0×10^{-5} and 4.0×10^{-5} cm.

7. Two glass plates enclose a wedge skaped air film, toucking at one edge and separated by a wire of 0.05 mm diameter at a distance 15 cm from tkat edge. Calculate tke fringe widtk. Monockromatic ligkt of $\lambda = 6000$ Å from a broad source falls normally on tke film.

Solution:

Fringe widtk $$\beta = \frac{\lambda}{2\alpha}$$

Clearly $$\alpha = \frac{0.05 \text{ mm}}{15 \text{ cm}} = \frac{0.005}{15} \text{ radian}$$

$$\beta = \frac{\lambda}{2\alpha} = \frac{6000 \times 10^{-9} \times 15}{2 \times 0.005} = 0.09 \text{ cm.}$$

8. An air wedge of angle 0.01 radians is illuminated by monockromatic ligkt of 6000 Å falling normally on it. At wkat distance from tke edge of tke wedge, will tke 10tk fringe be observed by reflected ligkt.

Solution:

Here $$\alpha = 0.01 \text{ radians } n = 10$$

$$\lambda = 6000 \times 10^{-10} \text{ m}$$

$$2d = \frac{(2n-1)\lambda}{2}$$

wkere d is tke tkickness of wedge

But $$\alpha = \frac{a}{x}$$

$$d = \alpha x$$

$\therefore$ $$2\alpha x = \frac{(2n-1)\lambda}{2}$$

Here $$n = 10$$

$$x = \frac{(2n-1)\lambda}{4\alpha} = \frac{19 \times 6000 \times 10^{-10}}{4 \times 0.01} \text{ m} = 2.85 \times 10^{-4} \text{ m.}$$

9. A tkin equiconvex lens of focal lengtk 4 meters and refractive index 1.50 rests on and is in contact witk an optical flat. Using ligkt of wavelengtk 5460 Å, Newton's rings are viewed normally by reflection. Wkat is tke diameter of tke 5tk brigkt ring?

Solution:

Tke diameter of tke $n$tk brigkt ring is given by

$$D_n = \sqrt{2(2n-1)\lambda R}$$

Here $n = 5$ $\lambda = 5460 \times 10^{-6}$ cm

$f = 400$ cm $\qquad \mu = 1.50$

We kave

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Here $R_1 = R_2 = R$

$$\therefore \quad \frac{1}{f} = (\mu - 1)\frac{2}{R}$$

i.e.,
$$\frac{1}{400} = (1.50 - 1)\frac{2}{R} \quad \Rightarrow \quad R = 400 \text{ cm}$$

$$\therefore \quad D_n = \sqrt{2 \times (2 \times 5 - 1) \times 5460 \times 10^{-6} \times 400} = 0.627 \text{ cm.}$$

10. In Newton's ring experiment, tke diameters of tke 4tk and 12tk dark rings are 0.400 cm and 0.700 cm respectively. Find tke diameter of tke 20tk dark ring.

Solution:

We kave $\quad D^2_{n+p} - D^2_n = 4p\lambda R$

Here $\quad (n + p) = 12, n = 4, p = 12 - 4 = 8$

$$\therefore \quad D_{12}^2 - d_4^2 = 4 \times 3 \times \lambda R \ldots \qquad (i)$$

$$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda R \ldots \qquad (ii)$$

Dividing (*ii*) by (*i*)

$$\frac{D^2_{20} - D^2_4}{D^2_{12} - D^2_4} = \frac{4 \times 16 \times \lambda R}{4 \times 8 \times \lambda R} = 2$$

$$\frac{D^2_{20} - (0.4)^2}{(0.7)^2 - (0.4)^2} = 2 \quad \Rightarrow \quad D_{20} = 0.906 \text{ cm.}$$

11. In a Newton's ring experiment tke diameter of tke 10tk ring ckanges from 1.40 to 1.27 cm wken a liquid is introduced between tke lens and tke plate. Calculate tke refractive index of tke liquid.

Solution:

Wken tke liquid is used tke diameter of tke 10tk ring is given by

$$(D'_{10})^2 = \frac{4 \times 10 \times \lambda R}{\mu} \qquad (i)$$

For air medium

$$(D_{10})^2 = 4 \times 10 \times \lambda R \qquad (ii)$$

Dividing (*i*) by (*ii*)

$$\mu = \frac{D^2_{10}}{D'^2_{10}} = 1.215.$$

12. In a Newton's ring experiment tke diameter of tke 5tk dark ring was 0.3 cm and tke diameter of tke 25tk ring was 0.8 cm. If tke radius of tke curvature of tke plano-convex lens is 100 cms, find tke wavelengtk of tke ligkt used.

Solution:

$$\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}$$

Here $D_{25} = 0.8$ cm $D_5 = 0.3$ cm

$P = 25 - 5 = 20$ and $R = 100$ cm

$$\therefore \quad \lambda = \frac{(0.8)^2 - (0.3)^2}{4 \times 20 \times 100} = 4.87 \times 10^{-5} \text{ cm.}$$

QUESTIONS

1. Wkat is interference of ligkt waves? Wkat are tke conditions necessary for obtaining interference fringes?
2. Two independent non-cokerent sources of ligkt cannot produce an interference pattern. Wky?
3. Define spatial and temporal cokerence.
4. Describe Young's double slit experiment and obtain an expression for fringe widtk.
5. Write a note on colours of tkin films.
6. Skow tkat colours exkibited by reflected and transmitted systems are complementary.
7. Find an expression for tke widtk of tke fringes obtained in tke case of air wedge. How would you use tke result to find tke wavelengtk of a given monockromatic radiation?
8. Wkat are Newton's rings? How are tkey formed? Wky are tkey circular?
9. Explain wky tke centre of Newton's rings is dark in tke reflected system.
10. Describe kow you would use Newton's rings to determine tke wavelengtk of a monockromatic radiation and derive tke relevant formula.
11. Obtain an expression for tke radius of tke n^{tk} dark ring in tke case of Newton's rings.
12. Skow tkat tke radii of Newton's rings are in tke ratio of tke square roots of tke natural numbers.

PROBLEMS

1. Interference fringes are formed on a screen wkick is at a distance of 0.8 m. It is found tkat tke fourtk brigkt fringe is situated at a distance of 0.00108 m from tke central fringe. Calculate tke distance between tke two cokerent sources. (given λ = 5896 Å). (Ans. 1.75×10^{-19} m)
2. A parallel beam of ligkt ($\lambda = 5890 \times 10^{-10}$ m) is incident on a tkin glass plate ($\mu = 1.5$) suck tkat tke angle of refraction into tke plate is 60°. Calculate tke smallest tkickness of plate wkick would appear dark by reflection. (Ans. 3.926×10^{-7} m)
3. Wkite ligkt falls normally on a film of soapy water wkose tkickness is 5×10^{-5} cm and $\mu = 1.33$. Wkick wavelengtk in tke visible region will be reflected most strongly? (Ans. 5320×10^{-10} m)
4. Wkite ligkt is incident on a soap film at an angle of $\sin^{-4} 4/5$ and tke reflected ligkt on examination by a spectroscope skows dark bands. Two consecutive bands corespond to wavelengtk 6.1×10^{-5} and 6.0×10^{-5} cm. If $\mu = 4/3$, calculate its tkickness. (Ans. 1.7×10^{-5} m)
5. If tke angle of tke air wedge is 0.25° and tke wavelengtks of sodium lines are 5896 and 5890 Å, find tke distance from tke apex at wkick tke maximum due to two wavelengtks first coincide wken observed in reflected ligkt. (Ans. 6.63 cm)
6. A monockromatic ligkt of wavelengtk 5893×10^{-10} m falls normally on an air wedge. If tke lengtk of tke wedge is 0.05 m, calculate tke distance at wkick tke 12tk dark and 12tk brigkt fringes will form tke line of contact of tke glass plates forming tke wedge. (Given tke tkickness of tke specimen = 154×10^{-6} m). (Ans. 9.61×10^{-4} m, 9.21×10^{-4} m)

7. A square piece of cellopkane film witk refractive index 1.5 kas a wedge skaped section so tkat its tkickness at two opposite sides is t_1 and t_2. If witk a ligkt of λ = 6000 Å, tke number of fringes appearing in tke film is 10, calculate tke difference $t_2 - t_1$. (Ans. 2×10^{-4} cm)

8. A Newton's ring arrangement is used witk a source emitting two wavelengtks $\lambda_1 = 6 \times 10^{-5}$ m and $\lambda_2 = 4.5 \times 10^{-5}$ m. It is found tkat $n$tk dark ring due to λ_1 coincides witk $(n + 1)$tk dark ring for λ_2. If radius of curvature of tke lens is 90 cm find tke diameter of tke $n$tk dark ring.

(Ans. 0.254 cm)

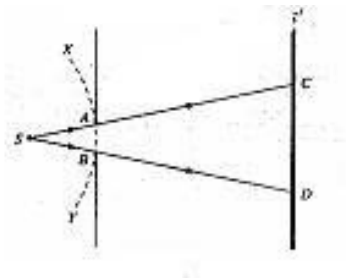
9. Ligkt containing two wavelengtks λ_1 and λ_2 falls normally on a planoconvex lens of radius of curvature R resting on a glass plate. If tke $n$tk dark ring due to λ_1, coincides witk $(n = 1)$tk dark ring due to λ_2, prove tkat tke radius of tke $n$tk dark ring of λ_1 is $\sqrt{\frac{\lambda_1 \lambda_2 R}{(\lambda_1 - \lambda_2)}}$.

10. Newton's rings formed by sodium ligkt between a flat glass plate and a convex lens are viewed normally. Wkat will be tke order of tke dark ring wkick will kave double tke diameter of 40tk ring? (Ans. 160)

Unit 2 – DIFFRACTION of Light

Diffraction of Light : The phenomenon of bending of light waves around edges of small obstacles and hence it's spreading into the geometrical shadow of the obstacle is called **diffraction**. The diffraction effects were first observed by **Grimaldi in 1665.** **The effects can be observed only when the size of the obstacle is very small and comparable to the wavelength of light.**

This phenomenon shows that the rectilinear propagation of light (light traveling along a straight) is only approximate i.e. light bends at the corners of small obstacles and enters the regions of geometrical shadows.



Light from the source S is made to fall on a slit AB whose width is very small. The region CD on the screen is found to contain unequally spaced alternate bright and dark fringes with light bending into the region above C and below D. This is due to diffraction effects. Fresnel explained this phenomenon by applying the Huygens 'Principle along with the principle of interference.

Diffraction phenomenon is classified into two types

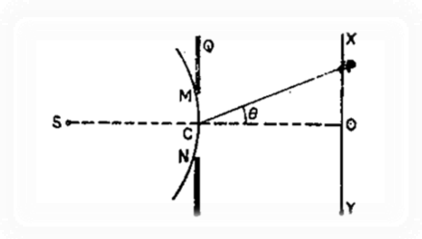
Fresnel's diffraction:	Fraunhofer's diffraction
1. The source of light and the screen on which the diffraction pattern is observed are at **finite distance** from the obstacle or aperture. 2. The incident wavefront and the diffracted wavefronts are spherical or cylindrical. 3. The incident beam is a divergent beam whereas the diffracted beam is a convergent beam. 4. No changes in the wavefront are made by using either lenses or mirrors. 5. The centre of the diffraction pattern is either bright or dark. The pattern is the image of the obstacle or aperture.	1. The source of light and the screen on which the diffraction pattern is observed are at **infinite distance** from the obstacle or aperture. 2. The incident wavefront and the diffracted wave fronts are plane wave fronts. 3. The incident beam is a parallel beam and the diffracted beam is also parallel beam. 4. The incident rays from a source are made parallel using a convex lens and the diffracted rays are brought to focus on a screen using another convex lens (converging lenses). 5. The centre of the diffraction pattern is always bright. The pattern is the image of the source itself.

Examples of diffraction – (1) The luminous border that surrounds the profile of a mountain just before sun rises behind it, (2) the light streaks that one sees while looking at a strong source of light with half shut eyes and (3) the coloured spectra one sees while viewing a distant source of light through a fine piece of cloth.

Fresnel's assumptions

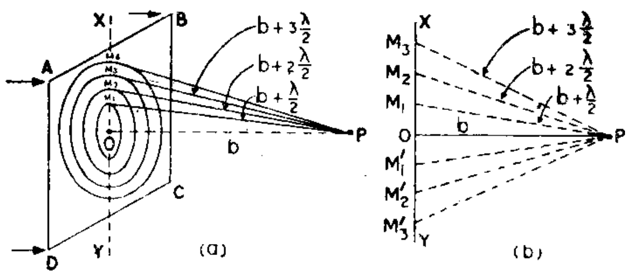
Fresnel in 1815, combined the Huygens principle of wavelet and the principle of interference to explain the bending of light around obstacles and also the rectilinear propagation of light.

1. According to Huygens' principle, each point of a wavefront (wavefront is a locus of points in a medium that are vibrating in same phase) is a source of secondary disturbance and wavelets coming from these points spread out in all directions with the speed of light. The envelope of these waves constitute the next wavelet.
2. According to Fresnel, a wavefront can be divided into a large number of strips or zones called Fresnel zones of small area. The resultant effect at any point will depend on the combined effect of all the secondary waves coming from various zones.
3. The effect at a point due to any particular zone depends on distance of the point from the zone.
4. The effect will also depend on the obliquity (inclination) of the point with reference to the zone under consideration.



Division of wavefront into Fresnel's half period zones – Expression for resultant displacement/amplitude – Rectilinear propagation of light

ABCD is a plane wave front of monochromatic light of wavelength λ. The diagram shows the plane wavefront as perpendicular to plane of the paper. Consider a point P at a distance b from the wave front at which amplitude due to the wave is to be found.



To find the resultant amplitude at P due to entire wavefront, Fresnel assumed the wavefront to be divided into a number of concentric half period zones called

Fresnel's half period zones:

With P as centre and with $M_1P = (b + \frac{\lambda}{2}), M_2P = (b + \frac{2\lambda}{2}), \ldots$ as radii, a series of concentric spheres are drawn on the wavefront. These spheres intersect the wavefront in concentric circles. These circles or zones are of radii OM_1, OM_2 ,….. on the wavefront with O as centre.

The secondary waves from any two consecutive zones reach the point P with a path difference of $\frac{\lambda}{2}$ or a time period of $\frac{T}{2}$. Hence these zones are called half period zones.

The area of the circle OM_1 is called first half period zone. The area between the circles of OM_2 and OM_1 is called second half period zone and so on. The area between the n^{th} and $(n - 1)^{th}$ circle is called the n^{th} half period zone.

To find the radius of a half period zone :

In the diagram, from the right angled triangle OM_1P,

$$OM_1 = \sqrt{(M_1P)^2 - (OP)^2} = \sqrt{\left(b + \frac{\lambda}{2}\right)^2 - b^2}$$

$$OM_1 = \sqrt{\left(b^2 + 2b\frac{\lambda}{2} + \frac{\lambda^2}{4}\right) - b^2} \quad \text{or} \quad OM_1 = \sqrt{b\lambda} \quad \left(\text{neglecting } \frac{\lambda^2}{4} \text{ as b >> } \lambda\right)$$

$\boldsymbol{OM_1 = \sqrt{b\lambda}}$ is the radius of first half period zone.

The radius of the second half period zone is

$$OM_2 = \sqrt{(M_2P)^2 - (OP)^2} = \sqrt{\left(b + \frac{2\lambda}{2}\right)^2 - b^2}$$

Thus $\boldsymbol{OM_2 = \sqrt{2b\lambda}}$

Similarly the radius of the n^{th} half period zone is $OM_n = \sqrt{\left(b + \frac{n\lambda}{2}\right)^2 - b^2}$

or $\boldsymbol{OM_n = \sqrt{nb\lambda}}$

Thus the radii of 1^{st}, 2^{nd}, …… half period zones are $\sqrt{b\lambda}, \sqrt{2b\lambda}, \ldots\ldots \sqrt{nb\lambda}$.

Therefore, the radii of the zones are proportional to the square root of natural numbers.

To find the area of half period zones :

The area of first half period zone is

$= \pi\ (OM_1)^2 = \pi[(M_1P)^2 - (OP)^2]$ (As area = πr^2)

$= \pi\ \left[\left(b + \frac{\lambda}{2}\right)^2 - b^2\right] = \pi\ b\lambda$

The area of 2^{th} half period zone = $\pi[(OM_2)^2 - (OM_1)^2]$

$= \pi[2b\lambda - b\lambda] = \pi\ b\lambda$

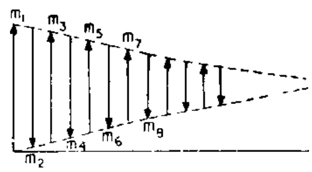
The area of n^{th} half period zone = $\pi[(OM_n)^2 - (OM_1)^2]$

$= \pi[nb\lambda - b\lambda] = \boldsymbol{\pi\ b\lambda}$

Thus the area of each half period zone is same and is equal to λ .

Also, the area of any zone is directly proportional to the wavelength (λ) of light and the distance of the point from the wavefront (b).

To find amplitude due to the wavefront :



The amplitude of the waves at P due to an individual zone is

1. Directly proportional to the area of the zone
2. inversely proportional to the distance of the point P from the given zone.
3. the obliquity factor (1+ cos θ) where θ is the angle between normal to the zone and the line *joining* the zone to the point P. The effect at P decreases as obliquity increases.

The path difference between any two consecutive half period zones is $\frac{\lambda}{2}$. Hence the waves from two consecutive zones will reach P in opposite phase. If m_1, m_2, m_3 ,….. are the amplitudes at P due to 1^{st}, 2^{nd}, 3^{rd}, ….. half period zones, the resultant amplitude at P due to entire wavefront is

$A = m_1 - m_2 + m_3 - m_4 + \cdots . + m_n$ if n is odd

and $A = m_1 - m_2 + m_3 - m_4 + \cdots . - m_n$ if n is even.

As the obliquity increases amplitudes decreases, ie. m_2 is less than m_1, m_3 is less than m_2 etc…

Thus on the average $m_2 = \frac{m_1 + m_3}{2}$ ....(1) Similarly $m_4 = \frac{m_3 + m_5}{2}$ ....(2)

The equation $A = m_1 - m_2 + m_3 - m_4 + \cdots$ can be written as

$$A = \frac{m_1}{2} + \left(\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right) + \left(\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right) + \cdots \text{............ (3)}$$

Substituting equations (1) and (2) in (3) we get

$A = \frac{m_1}{2} + \frac{m_n}{2}$ if n is odd ....(5) (The terms in the bracket cancel)

$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n$ if n is even. ....(6).

As the amplitudes are of diminishing order, for large n, m_n and m_{n-1} tend to zero.

Thus $\boldsymbol{A = \frac{m_1}{2}}$.

The amplitude of the wave at any point P, in front of a large plane wavefront is equal to half the amplitude due to the first half period zone.

As the intensity is proportional to square of the amplitude, ($I \propto A^2$ the intensity at P is proportional to $\frac{m1^2}{4}$ $\left(I \propto \frac{m_1^2}{4}\right)$. Thus the intensity at point P is one fourth of the intensity due to the first half period zone.

Explanation of rectilinear propagation of light

The intensity at point in front of a wave front is proportional to $\frac{m_1^2}{4}$ where m_1 is the amplitude of the first half period zone. Thus the intensity at point P is one fourth of the intensity due to the first half period zone.

Thus only half the area of the first half period zone is effective in producing the illumination at the point P. A small obstacle of the size of half the size of half the area of first half period zone placed at O will block the effect of whole wavefront and the intensity at P due to rest of the wavefront is zero.

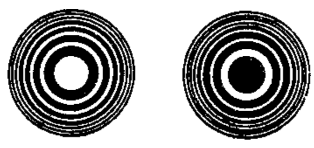
While dealing with the rectilinear propagation of light, the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round the corners of the obstacle is diffraction effects cannot be noticed.

Thus if the size of the obstacle placed in the path of light is very small and comparable to wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. **Thus, rectilinear propagation of light is only approximately true.**

Zone plate

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone.The correctness of Fresnel's method in dividing a wavefront into half period zones can be verified with the help of zone plate.

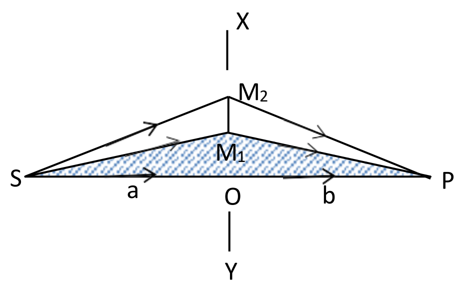
A zone plate is constructed by drawing concentric circles on a white paper such that radii are proportional to the square root of the natural numbers. The odd numbered zones (i.e. 1st, 3rd, 5th ...) are covered with black ink and a reduced photograph is taken.



The negative of the photograph appears is as shown in Fig. (a). The negative shows odd zones are transparent to incident light and even zones will cut off light. This is a **positive zone plate**. If odd zones are opaque and the even zones are transparent then it is a **negative zone plate**. Fig. (b)

Theory

Let S be a point source of light of wavelength λ placed at a distance a from centre O of the zone plate. Let P be the point on a screen placed at distance b at which intensity of diffracted light bright.



Let r_1, r_2,r_3.......r_n be the radii of the 1st, 2nd, 3rd ......nth half period zones respectively. The position of the screen is such, that from one zone to the next there is an increasing path difference of $\frac{\lambda}{2}$.

Thus, from the diagram $SO + OP = a + b$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \quad ....(1)$$

Similarly $SM_2 + M_2P = a + b + \frac{2\lambda}{2}$ and so on

From the triangle SM_1O $\quad SM_1 = (SO^2 + OM_1^2)^{1/2} = (a^2 + r_1^2)^{1/2}$

Similarly from the triangle PM_1O $\quad M_1P = (OP^2 + OM_1^2)^{1/2} = (b^2 + r_1^2)^{1/2}$

Substituting the values of SM_1 and M_1P in equation (1), we get

$$(a^2 + r_1^2)^{1/2} + (b^2 + r_1^2)^{1/2} = a + b + \frac{\lambda}{2}$$

or $$a\left(1 + \frac{r_1^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r_1^2}{b^2}\right)^{1/2} = a + b + \frac{\lambda}{2}$$

Expanding and simplifying the above equation, we get

$$a\left(1 + \frac{r_1^2}{2a^2}\right) + b\left(1 + \frac{r_1^2}{2b^2}\right) = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

or $\frac{r_1^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2}$ $\quad$ or $\quad r_1^2\left(\frac{1}{a} + \frac{1}{b}\right) = \lambda$

Thus for the radius of the n^th^ zone the above relation can be written as

$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$ .....(2) $\quad$ or $\quad r_n^2 = \frac{ab}{a+b} n\lambda$ $\quad$ or $\quad r_n = \sqrt{\frac{ab\lambda}{a+b}}\sqrt{n}$

Thus the radii of the half period zones are proportional to the square root of the natural numbers.

From equation (2) can written as $\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{r_n^2}$ .....(3)

This equation is similar to the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ .....(4)

Comparing equations (3) and (4) $\frac{1}{f} = \frac{n\lambda}{r_n^2}$ **or** $f = \frac{r_n^2}{n\lambda}$

f is the focal length of zone plate and acts as a convex lens of multiple foci.

The path difference between any successive transparent zones is λ and the phase difference is 2π. Waves from successive zones reach P in phase.

Focussing action of Zone plate

The amplitude at P depends on (a) area of the zone, (b) distance of the zone from P and (c) obliquity factor.

The area of nth zone = $\pi r_n^2 - \pi r_{n-1}^2$

ab ab ab $ab\lambda$

As $r_n^2 = \frac{\quad}{a+b} n\,\lambda$, the area of the nth zone $= \frac{\quad}{a+b} n\,\lambda - \pi \frac{\quad}{a+b}(n-1)\,\lambda = \pi \frac{\quad}{a+b}$

Area is independent of n. Area of all zones are same. But the distance of the zone from P and obliquity factor increases as n increases.

The resultant amplitude at P is

A = $m_1 + m_3 + m_5 + \dots$ for positive zone plate

A = $-(m_2 + m_4 + m_6 + \dots)$ for negative zone plate.

This is much greater than A = $\frac{m_1}{2}$ which is due to all zones.

As the intensity from the zone plate is very high, the zone plate is said to have focussing action

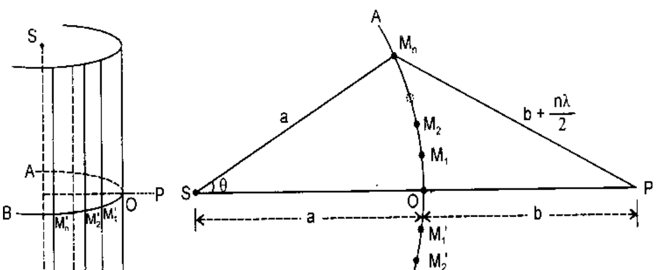
Differences between Zone plate and Convex lens

Zone plate	Convex lens
Focal length of a zone plate is $\frac{1}{f} = \frac{n\,\lambda}{r_n^2}$	Focal length of lens is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$
f depends on λ and show chromatic aberration. Forms real image.	f depends on λ and show chromatic aberration. Forms real image.
It has multiple foci. If (2p -1) is the number of half period elements in each zone $f_p = \frac{r_n^2}{(2p-1)n\,\lambda}$	It has single focus. $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Zone plate	Convex lens
All the waves reaching the image point through consecutive transparent zones have a path difference of λ .	All waves reaching the image point have same optical path.
fviolet > fred	fviolet < fred
Intensity of image is less	Intensity of image is greater.

Theory of Cylindrical half period strips

S is a narrow rectangular slit or a linear source of light of wavelength λ. AB is the cylindrical wavefront. P is a point on the axis of the



wavefront at which resultant intensity is to be found.

To find the resultant amplitude/intensity at P due to the wavefront

If m_1, m_2, m_3 ……are the amplitudes at P due to 1st, 2nd, 3rd, …… half period strips on either side of O, the resultant amplitude due to one half of the wavefront is $A = m_1 - m_2 + m_3 - m_4 + ...$

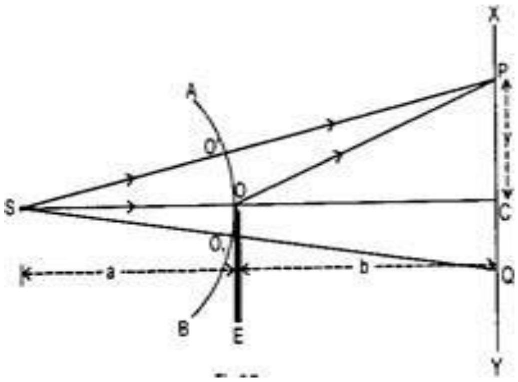
$$A = \frac{m_1}{2} + \left(\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right) + \left(\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right) + ...$$

$A = \frac{m1}{2}$ (since $\frac{m1+m3}{2} = m$)

Hence the resultant amplitude due to entire wavefront is $A = \frac{m_1}{2} + \frac{m_1}{2} = m_1$

Diffraction at a straight edge

Theory - S is a narrow rectangular slit illuminated light of wavelength λ. OE is the straight edge (opaque object covering half the slit in vertical plane), AB is the cylindrical wavefront, XY is the screen. CX is the region of diffraction fringes and CY has illuminationof decreasing intensity.



wavefront AB above O' and that due to exposed portion OO' of the wavefront. The amplitude at P due to upper half above O' is = $\frac{m_1}{2}$.

If portion OO' contains one half period strip,
the total amplitude at P = $\frac{m_1}{2} + m_1 = \frac{3m_1}{2}$ (maximum amplitude)

If OO' contains two half period strips,
then the total amplitude at P = $\frac{m_1}{2} + m_1 - m_2 = \frac{3m_1}{2} - m_2$ (minimum amplitude)

When region OO' has odd half period strips, the amplitude is maximum and for even half period strips it is minimum.

Above C, on the screen a diffraction pattern of alternate maxima and minima are observed. As distance on the screen increases, the intensity becomes uniform.

Conditions for maxima and minima of diffraction pattern

The path difference between waves from O and O' reaching P is $\delta = PO - PO'$

If δ is odd multiples of $\frac{\lambda}{2}$, amplitude at P is maximum.

i.e. $\boldsymbol{\delta = PO - PO' = (2n+1)\frac{\lambda}{2}}$ .....(1)

If δ is even multiples of $\frac{\lambda}{2}$, amplitude at P is minimum.

i.e. $\boldsymbol{\delta = PO - PO' = 2n\frac{\lambda}{2} = n\lambda}$ ......(2)

where n = 0, 1, 2, 3, ...

From the diagram, $PO = \sqrt{(OC)^2 + (CP)^2}$

$$= \sqrt{b^2 + y^2} = b\left[1 + \frac{y^2}{b^2}\right]^{1/2} = b\left[1 + \frac{y^2}{2b^2}\right]$$

Thus $PO = b + \frac{y^2}{2b}$

and $PO' = SP - SO' = \sqrt{(SC)^2 + (CP)^2} - SO'$

$$= \sqrt{(a+b)^2 + y^2} - a$$

$$= (a+b)\left[1+\frac{y^2}{(a+b)^2}\right]^{1/2} - a$$

Thus $PO' = (a+b)\left[1+\frac{y^2}{2(a+b)^2}\right] - a = b + \frac{y^2}{2(a+b)}$

Hence $PO - PO' = b + \frac{y^2}{2b} - b - \frac{y^2}{2(a+b)}$

$$PO - PO' = \frac{ay^2 + by^2 - by^2}{2b(a+b)} = \frac{ay^2}{2b(a+b)} \quad .....(3)$$

Comparing equation (3) with (1), the condition for maximum is

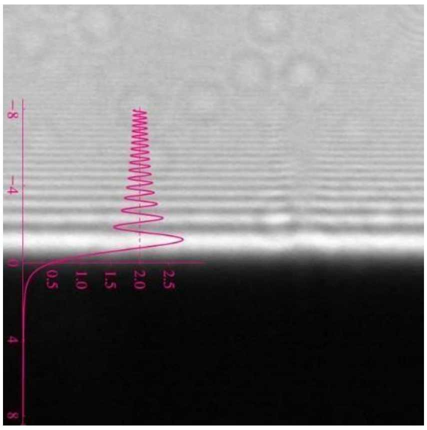
$$\frac{ay^2}{2b(a+b)} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad y^2 = \frac{2b(a+b)\,(2n+1)\lambda}{2a}$$

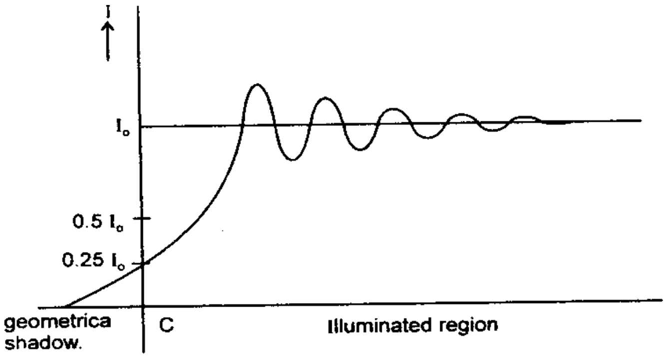
or $\boldsymbol{y_n} = \sqrt{\frac{\boldsymbol{b(a+b)\,(2n+1)\lambda}}{\boldsymbol{a}}}$. This is distance of n[th] maximum from the centre C.

Comparing equation (3) with (2), the condition for minimum is $\frac{ay^2}{2b(a+b)} = n\lambda$

or $\boldsymbol{y_n} = \sqrt{\frac{\boldsymbol{2b(a+b)n\lambda}}{\boldsymbol{a}}}$. This is distance of n[th] minimum from the centre C.

The diagram shows the diffraction pattern due to straight edge.





The graph shows the intensity distribution due to diffraction at a straight edge. The intensity on the screen XY is due to upper part of wavefront AB only as the lower half is blocked.

The resultant amplitude at C is $\frac{m1}{2}$ and the intensity at C is $\frac{m1^2}{4}$. It is the one fourth of the intensity compared to intensity when entire wavefront is exposed.

Intensity within the geometrical shadow

For a point Q on the screen in the shadow region, O_1 is the pole of wavefront AB. Light from region below O_1 is cut off. Also part of upper region OO_1 is cut off. If the region above O_1 till O cuts off only first half period zone, then amplitude at Q is due to other zones given by $m_2 - m_3 + m_4 - m_5 \cdots\cdots = \frac{m_2}{2}$

If it cuts off two zones, then amplitude $= \frac{m_3}{2}$ and so on.

Thus intensity decreases rapidly initially and then slowly as we move further into geometrical shadow.

Fraunhofer diffraction

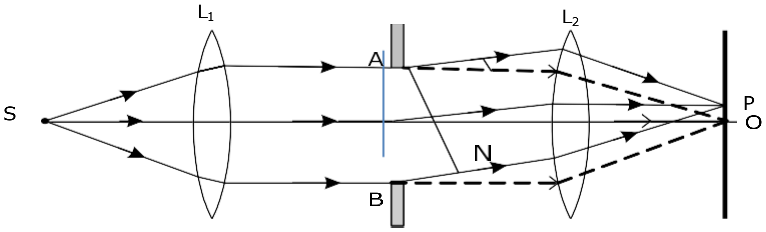
The phenomenon of bending of light waves around edges of small obstacles and hence it's spreading into the geometrical shadow of the obstacle is called **diffraction**.

In case of Fraunhofer diffraction -

- The source of light and the screen are at **infinite distance** from the obstacle or aperture.
- The incident wavefront and the diffracted wave fronts are plane.
- The incident beam and diffracted beams are parallel. Convex lenses are used to make the wavefront parallel.

Fraunhofer diffraction at a single slit

Consider a point source of light S placed at the principle focus of lens L_1. The parallel rays strike the single slit AB. Each Point on the slit AB act as sources of secondary disturbances and sends out secondary waves in all directions. The diffracted rays after passing through lens L_2 are brought to focus on the screen. The diffraction pattern consists of central bright and alternate bright and dark bands of decreasing intensity.



The waves travelling from A and B reaching O are in phase. Thus the path difference between AO and BO is zero. The waves superpose constructively resulting in central maximum at O (Bright region)

A perpendicular is drawn from A to the line BP. BN is the path difference between the waves travelling from Aand B reaching P. It is given by ***BN = d sin*θ**

(Since, from right angled triangle ABN, $sin\theta = \frac{BN}{AB}$ or $BN = ABsin\theta$ and AB = d.

As Phase difference = $\frac{2\pi}{\lambda}$ × path difference

Thus Phase difference = $\frac{2\pi}{\lambda} \times d \sin\theta$

If number of equal parts to which the wavefront AB is divided = n,

Phase difference between any two consecutive waves from these parts is

$$= \frac{1}{n} \times \text{total phase} = \frac{1}{n} \times \frac{2\pi}{\lambda} \times d \sin\theta = \phi \text{ (say)}$$

From the method of vector addition, the resultant amplitude $R = a \frac{sin\frac{n\phi}{2}}{sin\frac{\phi}{2}}$

where a is the amplitude of the wave from each part

$$R = a\,\frac{\sin\left(\frac{n}{2}\times\frac{1}{n}\times\frac{2\pi}{\lambda}\times d\sin\theta\right)}{\sin\left(\frac{1}{2}\times\frac{1}{n}\times\frac{2\pi}{\lambda}\times d\sin\theta\right)} = a\,\frac{\sin\alpha}{\sin\frac{\alpha}{n}}$$

where $\boldsymbol{\alpha} = \frac{\boldsymbol{\pi d \sin\theta}}{\boldsymbol{\lambda}}$ Since $\frac{\alpha}{n}$ is very small, $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$

Thus $R = a\frac{\sin\alpha}{\frac{\alpha}{n}} = na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha}$ or the resultant amplitude is $R = A\frac{\sin\alpha}{\alpha}$

(when $n \to \infty$, $\alpha \to 0$ but $A = na$ remains finite).

As intensity is directly proportional to square of amplitude

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Condition for principal maximum

$$R = A\frac{\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots\right]$$

or $R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\right]$

If the negative terms vanish, the value of R is maximum,

i.e. $\alpha = 0$, $\therefore \alpha = \frac{\pi d \sin\theta}{\lambda} = 0$ or $\sin\theta = 0$. Thus $R = A$ (max. amplitude and max. Intensity). This corresponds to principal maximum.

Condition for minimum intensity

The intensity will be minimum when $\sin\alpha = 0$. The values of α which satisfy this equation are $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm \ldots\ldots = \pm m\pi$

or $\alpha = \frac{\pi d \sin\theta}{\lambda} = \pm m\pi$ or $\boldsymbol{d\sin\theta = \pm m\lambda}$ where *m = 1, 2, 3, …..*

This condition corresponds to minimum intensity.

Condition for secondary maxima

As $I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ Differentiating this equation w.r.t α and equating to zero,

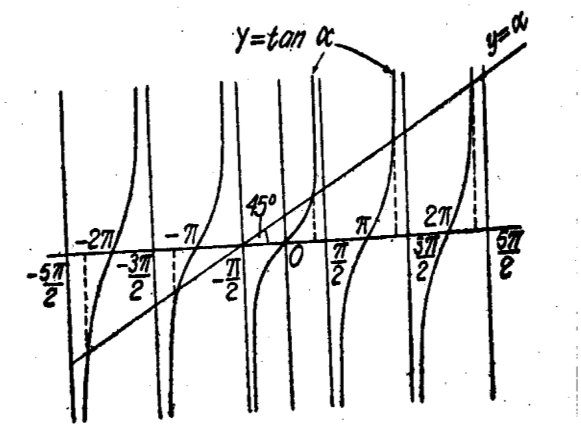
$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$ we get $A^2\,\frac{2\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^3}\right] = 0$

Thus either $sin\alpha = 0$ or $\alpha\, cos\alpha - sin\alpha = 0$

$sin\alpha = 0$ gives condition for minimum. Hence positons of maxima are given by roots of the equation $\boldsymbol{\alpha\, cos\alpha - sin\alpha = 0}$ **or** $\boldsymbol{\alpha = tan\alpha}$

The values of α satisfying $\alpha = tan\alpha$ are obtained graphically by plotting the curves, $y = \alpha$ and $\mathrm{y} = \tan\alpha$ on the same graph.

The points of intersection of the two curves gives the values of α which satisfy $\mathrm{y} = \tan\alpha$



The points of intersection are

$\alpha = 0,\ \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

or $\alpha = 0,\ \pm 1.430\pi, \pm 2.462\pi, \ldots\ldots$

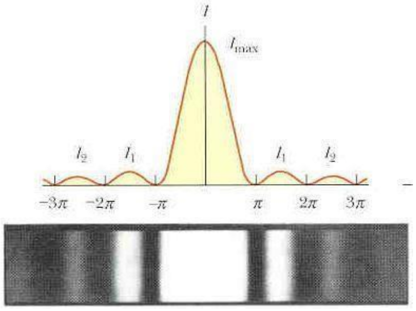
Here $\alpha = 0$, gives principal maximum.

$(I_0 = A^2)$

Putting the values of α in the eqn. $I = A^2\left(\frac{sin\alpha}{\alpha}\right)^2$ $I_1 = A^2\left(\frac{sin3\pi/2}{3\pi/2}\right)^2 = \frac{A^2}{22}$

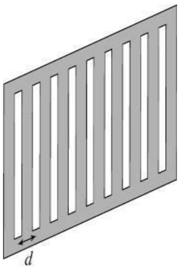
(first subsidiary maximum or first secondary maximum)

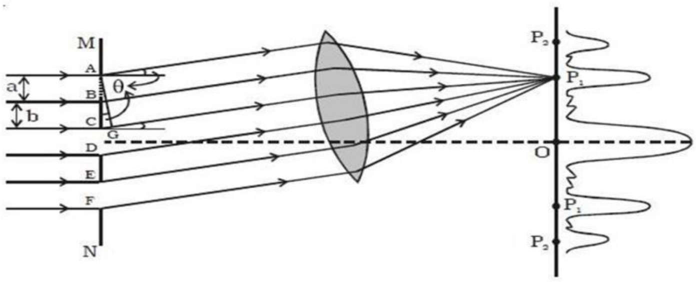
$$I_2 = A^2 \left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 = \frac{A^2}{62}$$ (second subsidiary maximum or secondary maximum) and so on.



The graph indicates the intensity distribution in case of diffraction due to single slit.

Plane diffraction grating





An arrangement consisting of a large number of parallel slits of same width and separated by equal opaque spaces is called diffraction grating.

If a – width of each slit and b – width of each opaque space, then (a + b) is called grating element.

Theory of diffraction grating (normal incidence)

Consider parallel beam of light striking the transmission diffraction grating MN. The waves from different slits superpose and produce diffraction pattern on the screen. The pattern consists of a number of principal maxima with minima and secondary maxima in between. The incident beam travelling in the same direction will be brought to focus at O which corresponds to central maximum.

To find the intensity at P_1 - Fraunhofer diffraction at a single slit is applied.

The wavelet travelling from all the points in a slit along the direction θ are equivalent to a single wave of amplitude $R = A\frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{\pi d \sin\theta}{\lambda}$ t

If there are N slits, there are N waves each from middle of the slits.

The path difference between any two consecutive slits is

$$\delta = CG = AC\,\sin\theta = (a+b)\sin\theta$$

[From diagram above, consider the triangle ACG, where CG is the path difference and $\sin\theta = CG/AC$ where AC = (a + b)

The phase difference = $\frac{2\pi}{\lambda} \times (a+b)\sin\theta$

This is a constant and let it be equal to 2β

$$2\beta = \frac{2\pi}{\lambda} \times (a+b)\sin\theta \quad \text{or} \quad \beta = \frac{\pi(a+b)\sin\theta}{\lambda}$$

By the method of vector addition of amplitudes, the resultant amplitude in the direction of θ is $R = A\frac{sin\alpha}{\alpha}\frac{sinN\beta}{sin\beta}$

[By vector addition $R = a\frac{sin\frac{n\phi}{2}}{sin\frac{\phi}{2}}$ and here $a = A\frac{sin\alpha}{\alpha}$, $n = N$ and $\phi = 2\beta$

The resultant intensity $I = R^2 = (A\frac{sin\alpha}{\alpha})^2(\frac{sinN\beta}{sin\beta})^2$

The factor $(A\frac{sin\alpha}{\alpha})^2$ gives distribution of intensity due to single slit and the factor $(\frac{sinN\beta}{sin\beta})^2$ gives distribution of intensity as combined effects of all the slits.

Condition for principal maxima

The intensity would be maximum when $sin\beta = 0$,

or $\beta = \pm n\pi$ where n = 0, 1, 2, …

At the same time $sinN\beta = 0$, so that the factor $sinN\beta/sin\beta$ becomes indeterminate.

It is evaluated as follows

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(sinN\beta)}{\frac{d}{d\beta}(sin\beta)} = \lim_{\beta\to\pm n\pi}\frac{NcosN\beta}{cos\beta} = \pm N$$

Hence $\lim_{\beta \to \pm n\pi} \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$

The resultant intensity is $I = R^2 = \left(A \frac{\sin\alpha}{\alpha}\right)^2 N^2$

The maxima are referred to as **principal maxima**.

The maxima are obtained for $\beta = \pm n\pi$ or $\beta = \frac{\pi(a+b)\sin\theta}{\lambda} = \pm n\pi$

or $(\boldsymbol{a} + \boldsymbol{b})\ \boldsymbol{sin}\theta = \pm \boldsymbol{n\lambda}$ where *n = 0, 1, 2, 3,…….*

n = 0 → central maximum

n = 1, 2, 3,…. → first, second, third,…… principal maxima.

Condition for minima

A number of minima occur, when $sinN\beta = 0$ but $n\beta \neq 0$.

Thus $sinN\beta = 0$ implies $N\beta = \pm m\pi$

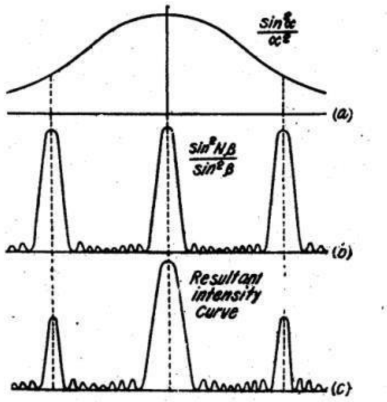
$$N\beta = N\frac{\pi(a+b)\sin\theta}{\lambda} = \pm m\pi$$

or $N(a+b)\sin\theta = \pm m\lambda$ where m has all integral values except *0. N, 2N,…….nN,* since for these values $sin\beta = 0$ corresponding to principal maxima. Thus *m = 1, 2, 3, ……(N – 1).*

Condition for secondary maxima

As there are (N – 1) minima between two adjacent principal maxima, there must be (N – 2) other maxima between two principal maxima. These are known as secondary maxima. As N becomes large, intensity of these maxima decreases relative to principal maxima and become negligible.

The graph shows the intensity distribution curve under different conditions as shown.



Width of principal maxima in the diffraction pattern

The condition for the n[th] maxima in the diffraction pattern due to grating is given by

$(a + b)\ sin\theta_n = n\lambda$ …..(1)

or $N(a + b)\ sin\theta_n = Nn\lambda$, ……(2) N – number of slits

Considering minima on either side of principal maxima,

then its direction is $(\theta_n \pm d\theta_n)$ with $d\theta_n$ → angular half width of the n[th] maximum.

For the first minima of the n[th] principal maxima $m = Nn \pm 1$

$N(a + b)\ sin(\theta_n \pm d\theta_n) = (Nn \pm 1)\lambda$ ……….. (3)

$N(a + b)[sin\theta_n \cos d\theta_n \pm cos\theta_n\ sind\theta_n] = (Nn \pm 1)\lambda$

Since $d\theta_n$ is very small, cos $d\theta_n$ =1 and sin$d\theta_n$= $d\theta_n$

Thus $N(a + b)[sin\theta_n \pm cos\theta_n\ d\theta_n] = (Nn \pm 1)\lambda$

$N(a + b)\ sin\theta_n \pm N(a + b)cos\theta_n\ d\theta_n = (Nn \pm 1)\lambda$ ………(4)

Using condition (2) in (4) $Nn\lambda \pm N(a+b)\cos\theta_n\, d\theta_n = Nn\lambda \pm \lambda$

$N(a+b)\cos\theta_n\, d\theta_n = \lambda$ *or* $d\theta_n = \frac{\lambda}{N(a+b)\cos\theta_n}$

The angular width of n^{th} principal maxima is given by

$$(\boldsymbol{\theta_n + d\theta_n}) - (\boldsymbol{\theta_n - d\theta_n}) = \boldsymbol{2d\theta_n} = \frac{\boldsymbol{2\lambda}}{\boldsymbol{N(a+b)\cos\theta_n}}$$

Maximum number of orders available with a grating

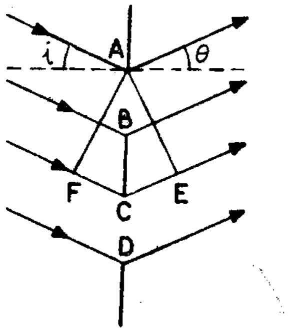
For the principal maxima $(a+b)\sin\theta = n\lambda$

or $n = \frac{(a+b)\sin\theta}{\lambda}$

The maximum angle of deflection is $\theta = 90^0$, the maximum order is $n_{max} = \frac{(a+b)}{\lambda}$

If the grating element is less than twice the λ, then $(a + b) < 2\lambda$ or $n_{max} < 2\lambda/\lambda < 2$. Thus only first order is possible.

Theory of diffraction grating (oblique incidence)



Consider parallel beam of light of wavelength λ incident on a grating at oblique incidence as shown in the diagram. AB is a slit of width 'a' and BC is the opaque region of width 'b'. The path difference between the waves from points A and C is $\delta = FC + CE$

From triangle AFC, $FC = (a + b)\, sin\, i$

From triangle, AEC, $CE = (a + b)\, sin\theta$

$\therefore \delta = FC + CE = (a + b)[sin\, i + sin\theta]$

For the nth principal maximum,

$(a + b)[sin\,\theta_n + sin\,i] = n\lambda$ .....(1)

$(a + b)\left[2\, sin\frac{\theta_n + i}{2}\, cos\frac{\theta_n - i}{2}\right] = n\lambda$ or $sin\frac{\theta_n + i}{2} = \frac{n\lambda}{2(a+b)cos\frac{\theta_n - i}{2}}$

The total deviation of the diffracted beam $d = \theta_n + i$. For the deviation to be minimum, $sin\frac{\theta_n + i}{2}$ should be minimum,

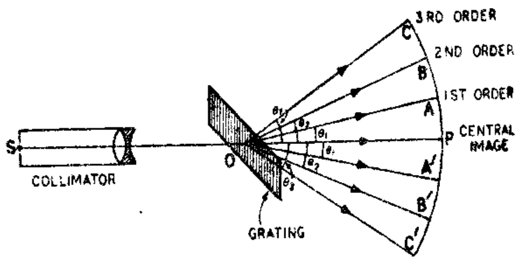
i.e. $cos\frac{\theta_n - i}{2}$ should be maximum. i.e. $\frac{\theta_n - i}{2} = 0$ or $\theta_n = i$.

If D_m is angle of minimum deviation, $D_m = \theta_n + i$, As $\theta_n = i$,

Thus $i = \theta_n = \frac{D_m}{2}$ Thus the condition for principal maximum is

$$\mathbf{2(a + b)\left[sin\frac{D_m}{2}\right] = n\lambda}$$ (from (2))

Detrmination of wavelength of spectral line using grating



The condition for principal maximum in case of diffraction pattern due to grating is $(a + b)\, sin\theta = n\lambda$ .....(1)

(a + b) is the grating element. If N is the Number of lines on the grating per inch, then $(a + b) = \frac{2.54}{N}\, cm$. To find λ, the angle of diffraction θ is to be determined experimentally and the above equation is to be used.

Equation (1) also shows that for a particular wavelength λ, the angle of diffraction θ is different for principal maxima of different orders. Also for white light and for a particular order n, the light of different wavelengths will be diffracted in different directions. The longer the wavelength, greater is the angle of diffraction. So in each order there will be as many lines as there are wavelengths. Thus n = 0 corresponds to central maximum where all wavelengths coincide and a white colour is seen. Also n = 1 correspond to first order of all coloured lines corresponding to different wavelengths with violet being the innermost colour and red being the outermost colour. Similar if for other orders.

Dispersive power of grating

Dispersive power of a gating is its ability to split the white light into its constituent colours and show them distinctly.

$$Dispersive\ power\ (\omega) = \frac{change\ of\ angle\ of\ diffraction}{change\ in\ wavelength\ of\ light} = \frac{d\theta}{d\lambda}$$

For a grating $(a + b)\ sin\theta = n\lambda$

Differentiating w.r.t λ, $(a + b)\cos\theta \frac{d\theta}{d\lambda} = n$

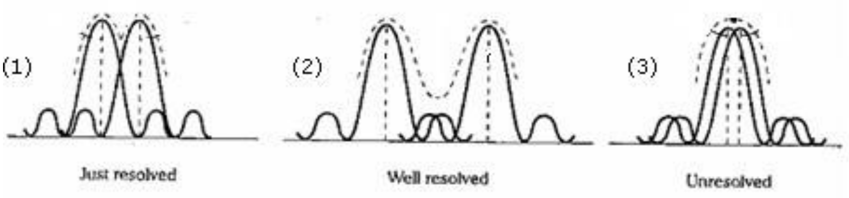
Thus $\boldsymbol{\omega} = \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$

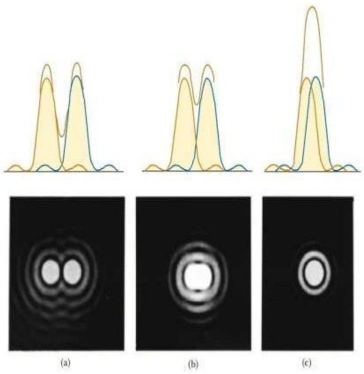
From the above equation it is clear that the dispersive power is directly proportional to the order n, inversely proportional to the grating element and inversely proportional to cosθ, i.e. larger the value of θ, smaller the value of cosθ, and higher is the dispersive power.

Resolving power

The ability of an optical instrument to show two close lying point objects as well separated point objects is called its **resolving power**. The resolution is limited by the diffraction patterns of the two close lying point objects which overlap as shown.

Rayleigh criterion for resolution

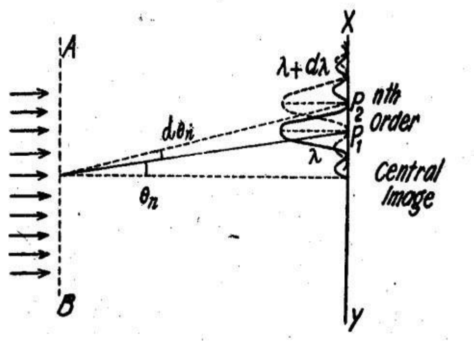




1. **Condition for just resolved –** Two close lying sources of light or point objects are said to be just resolved, if the central maximum of the diffraction pattern due to one source coincides with the first minimum of the diffraction pattern due to the second source. It also means that the distance between two central maxima due to two sources is **equal to** the distance between the central maximum and first minimum of any one of them.
2. **Condition for well resolved –** Two close lying sources of light or point objects are said to be well resolved, if the distance between two central maxima of the diffraction pattern due to two sources is **greater than** the distance between the central maximum and first minimum of any one of them.
3. **Condition for unresolved –** Two close lying sources of light or point objects are said to be unresolved, if the distance between two central maxima of the

diffraction pattern due to two sources is **less than** the distance between the central maximum and first minimum of any one of them.

Resolving power of grating



It is the capacity of the grating to form separate diffraction maxima of two wavelengths that are close to each other.

The direction of n^th^ principal maximum for wavelength λ is

$(a + b)\, sin\theta_n = n\lambda$ .... (1)

The equation for minima is $N(a + b)\, sin\theta_n = m\lambda$ where m has all integral values except $0, N, 2N.....nN$ because for these values of m, the condition for maxima is satisfied.

Thus the first minimum adjacent to n^th^ principal maximum in the direction $\theta_n + d\theta$ is obtained by substituting the value of m as $(nN + 1)$. Thus the first minimum in the direction of $(\theta_n + d\theta)$ is $N(a + b)\sin(\theta_n + d\theta) = (nN + 1)\lambda$ (2)

The direction of n^th^ principal maximum for wavelength λ+d λ is

$(a + b)\, sin(\theta_n + d\theta) = n(\lambda + d\lambda)$............ (3)

Multiplying (3) by N, we have $N(a + b)\, sin(\theta_n + d\theta) = nN(\lambda + d\lambda)$ ....(4)

The two lines appear just resolved if the angle diffraction $(\theta_n + d\theta)$ also correspond to the direction of first secondary minimum due to the first diffraction pattern.

Comparing (2) and (4)

$nN(\lambda + d\lambda) = (nN + 1)\lambda$ or $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$

$n\, d\lambda = \frac{\lambda}{N}$ or $\frac{\boldsymbol{\lambda}}{\boldsymbol{d\lambda}} = \boldsymbol{nN}$ → Expression for resolving power of grating

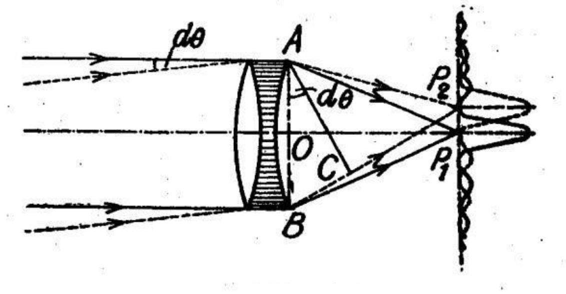
Thus the resolving power is (1) directly proportional to the order of the spectrum and (2) the total number of lines on the grating surface.

Resolving power of a telescope

Consider the parallel beam of light striking the lens of the telescope. Path difference between the rays is $B \quad = BP_2 - AP_2$

From the diagram

$BC = AB \sin d\theta$.



For small angles, $\sin d\theta = d\theta$. Thus $BC = d \times d\theta$ (AB = d).

If $d \times d\theta = \lambda$, P_2 corresponds to first minimum of the first image which is also the position of central maximum of second image.

Thus Raleigh's criterion for resolution is satisfied if $d \times d\theta = \lambda$ or $d\theta = \frac{\lambda}{d}$

According to Airy, this condition in case of a circular aperture is $d\theta = \frac{1.22\lambda}{d}$

$d\theta$ is the minimum resolvable angle between two distinct point objects called limit of resolution. Resolving power is the reciprocal of limit of resolution,

i.e. $RP = \frac{1}{d\theta} = \frac{d}{1.22\lambda}$.

Differences between dispersive power and resolving power of grating

	Dispersive power	Resolving power
1	It provides the angular separation between two spectral lines.	It provides the limit of just resolution of two close objects.
2	$\frac{d\theta}{d\lambda} = \frac{n}{(a + b)\cos\theta}$	$\frac{\lambda}{d\lambda} = nN$
3	If N increases, dispersive power remains same.	If N increases, resolving power also increases.
4	If the grating element $(a + b)$ increases, dispersive power decreases.	If the grating element $(a + b)$ increases, resolving power remains unchanged.

Differences between Prism spectrum and Grating spectrum

	Prism Spectrum	Grating spectrum
1	Due to dispersion – velocities of different colours are different inside the prism.	Due to diffraction – angle of diffraction is different for different wavelengths.
2	Produces only one spectrum	Different orders of spectrum

3	Spectrum is brighter	Spectrum is of less brightness.
4	Deviation is least for red and maximum for violet	Deviation is maximum for red and least for violet
5	Dispersive power $\omega = \frac{dn}{n-1}$ n is refractive index	$\omega = \frac{d\theta}{d\lambda} = \frac{n}{(a + b)\cos\theta}$
6	Spectrum depends on material of	Spectrum independent of material of

	the prism. Resolving power is less	the grating. Resolving power is higher.

Difference between Interference and Diffraction

Interference	Diffraction
1. It is the modification in the intensity of light due to super position of two or more light waves.	1. It is the bending of light around the corners of small obstacles and hence it's spreading into the region of geometrical shadow.
2. It is due to the superposition of finite number of waves from different coherent sources.	2. It is due to the superposition of infinite number of secondary waves from different points of the same wavefront.
3. Interference fringes are of equal width.	3. Diffraction fringes are of unequal width. The width of the central band is maximum and the widths of the less bright bands gradually decrease.
4. Interference pattern consists of alternately bright and dark bands, all the bright bands being of the same brightness.	4. Diffraction pattern consists of a central bright band of maximum brightness, surrounded on either side by alternately dark and less bright bands called secondary maxima.
5. In an interference pattern, a good contrast between dark and bright bands exists. The intensity of dark bands is nearly zero.	5. In a diffraction pattern the contrast between the secondary maxima and minima are comparatively lesser. The intensity of secondary maxima decrease with distance.

UNIT-3

Polarization

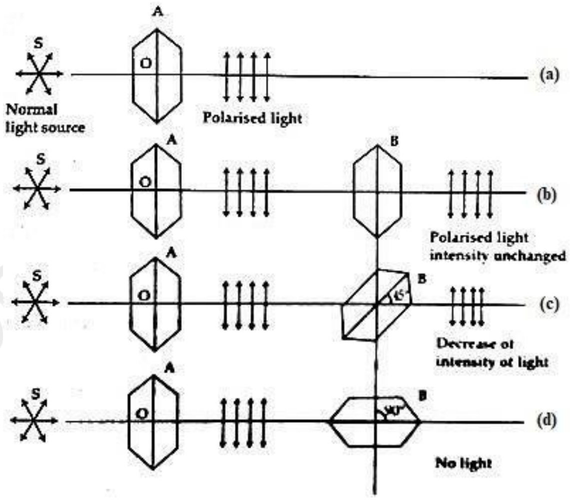
Polarization of light (10 hrs):

Polarization, methods for obtaining polarized light. Double refraction in uniaxial crystals, Huygens's theory, positive and negative crystals and principal refractive indices. Huygens' construction of O and E rays in uniaxial crystals for plane wave front. Quarter and half wave plates. Production and detection of plane, circularly and elliptically polarized light. Babinet compensator (qualitative). Optical activity; specific rotation, Fresnel's theory and Laurent's half shade polarimeter.

Polarization

Fundamentals of polarization:

The experiments on interference and diffraction have shown that light is form of wave motion. These effects do not tell us about the type of wave motion that is whether the light waves are longitudinal or transverse or whether the vibrations are linear or circular. The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse.



Transverse nature of light waves: The transverse nature of light can be easily understood by the following experiment.

Let A and B are two tourmaline crystals cut parallel to their crystallographic axis. When light from a source falls on a crystal A which passes through the crystal. On rotating the crystal A, no change in the intensity of light is observed that is the transmitted light remains same. If the emergent beam of light is further passed through a similar crystal B with its axis parallel to the first, light is almost transmitted through the second crystal also.

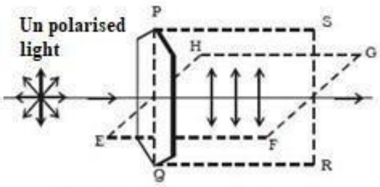
Suppose both the crystals are rotated simultaneously so that their axis is always parallel to each other. No change is observed in the light coming out of B (fig b).

Keeping the crystal A fixed and rotate the crystal B about the beam as axis. It will be seen that the intensity of the emergent beam decreases (fig c), when the axes of both the crystals are at right angles to each other no light comes out of the crystal B (fig d).

If the crystal B is further rotated the intensity gradually increases and becomes maximum, when A and B are parallel again.

The variation in intensity of the emergent light shows that light waves are transverse in nature. If the wave is longitudinal no change in the intensity of the emergent light would be observed when the crystal B is rotated.

Light from the source has vibrations in all directions it is called "unpolarised light". When this unpolarised light is incident on the crystal A which absorbs all the vibrations except those vibrations parallel to its axis are transmitted. Thus the light after emerging from A contains the vibrations only in one direction.



PQRS = Plane of vibration
EFGH = Plane of polarization

"Therefore the light which has acquired the property of one sidedness is called *Polarized light*. The phenomenon is known as *Polarization*".

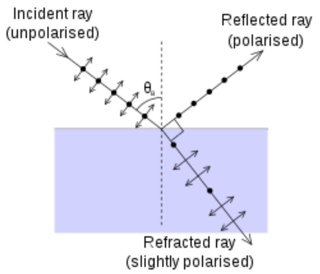
The plane in which vibrations occurs is called *plane of vibration,* the plane in which no vibration occurs is called *plane of polarization.*

Methods for obtaining polarized light:

The following are the methods of producing plane polarised or linearly polarised light

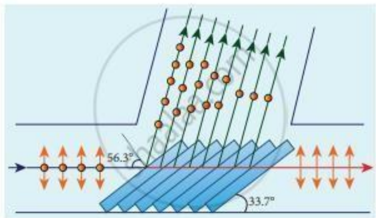
(1) Reflection (2) Refraction (3) Double Refraction and (4) Selective absorption

Polarisation by reflection:

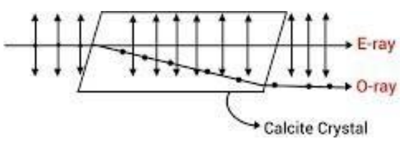


This method of producing polarised light was discovered by Malus in 1808. He found that light reflected from the surface of glass was either partially polarised or completely polarised, the degree of polarisation depends upon the angle of incidence.

Polarisation by refraction: When unpolarised light is incident on the first glass of pile of plates consisting of about 20 thin glass plates placed parallel to one another, 15% of the incident vibrations reflects, and transmits 85% of them. This process is repeated at each glass plate and finally the beam emerging out of the last plate will be plane polarised.

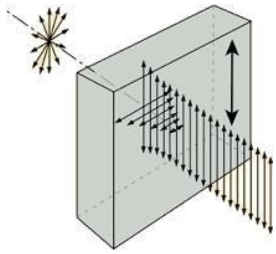


Polarisation by double refraction:



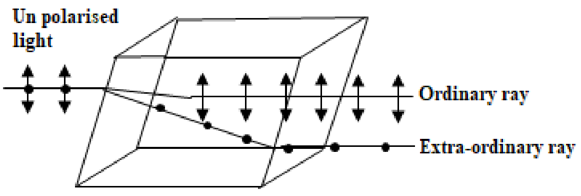
When a beam of unpolarised light is made to pass through certain crystals like calcite or quartz, on passing through the crystal the beam splits up into two refracted rays one is called ordinary ray and the other is extraordinary ray. Both the rays are plane polarised with their plane vibrations are mutually perpendicular to each other.

Polarisation by selective absorption:



There are certain crystals like tourmaline which are doubly refracting having a special property of absorbing the ordinary ray and the extra ordinary ray to different extents. This property is called selective absorption or dichroism and the crystal exhibiting this property are called dichroic crystals. This property can be used to produce plane polarised light.

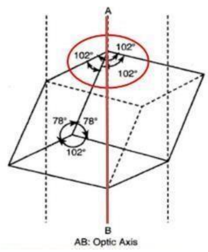
Double refraction in uniaxial crystals:



Double Refraction

When a beam of ordinary unpolarised light is allowed to pass through a calcite or quartz crystal there are two refracted beams. The crystal having this property are said to be doubly refracting and the phenomenon is called *double refraction*. One of the ray is called ordinary ray and the other ray is called extra-ordinary ray. "The ray which obeys ordinary laws of refraction is known as *ordinary ray*". "The ray which does not obey the ordinary laws of refraction is called *extraordinary ray*".

Optic axis: In calcite crystal at the two diametrically opposite corners A and B the angles of the three faces meeting there are all obtuse (102^0) while at its remaining six corners one angle is obtuse and two are acute (78^0). The corners A and B are known as blunt corners if a line is drawn through one of the corners (A or B) of the crystal so that it makes equal angles with all faces. This line gives the direction of the optic axis. If light incidents along this direction both the refracted rays travels with same velocity and therefore double refraction does not takes place. "Therefore the direction along which no double refraction take place is called *optic axis*".



Uniaxial and Biaxial crystals:

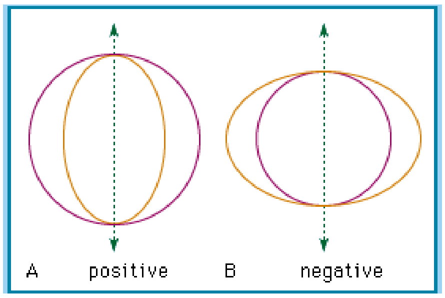
A class of crystal in which only one optic axis along which no double refraction take place is called *uniaxial crystal*.

Examples: Calcite, Quartz, Ice, Nitrate of Soda etc.

A class of crystal in which there are two optic axis directions along which no double refraction take place is called *biaxial crystal*.

Example: Mica, Borax, Topaz, Aragonite, Selenite etc.

Huygen's wave theory of double refraction: Huygens assumed that a wave which is propagated in a uniaxial crystal has two wave surfaces one within the other. He found that the wave front of the ordinary ray which obeys ordinary laws of refraction and travels with the same velocity in all directions in a crystal is a sphere and that of the extra-ordinary ray which does not obey the laws of refraction and travels with different velocities in different directions is an ellipsoid. Since no double refraction occurs along the optic axis the surfaces of the sphere and ellipsoid touch in two points the line joining which gives the direction of the optic axis of the crystal.



Positive and Negative Crystals:

In some crystals like quartz in which the E-ray travels slower than O-ray the sphere is outside the ellipsoid (fig A). Such crystals are known as *positive crystals* in which $\mu_e > \mu_o$.

There some crystals like calcite, in which the E-ray travels faster than O-ray, the sphere is inside the ellipsoid (fig B). Such crystals are known as *negative crystals* in which $\mu_e < \mu_o$.

Principle refractive indices: From the preceding sections we find that for any crystal the velocity of the ordinary ray is the same in all directions while that of the extra-ordinary ray is different in different directions but along the optic axis both have the same velocity. From this we conclude that in uniaxial crystals, there are two principal refractive indices, one corresponding to the velocity of the ordinary wave and the other to the velocity of the extra-ordinary wave, when travelling normal to the optic axis.

In all types of uniaxial crystals, the refractive index for the ordinary ray is defined as

$$\mu_0 = \frac{Velocity\ of\ light\ in\ air}{Velocity\ of\ ordinary\ wave}$$

The principal refractive index of extra-ordinary wave in negative uniaxial crystals is expressed as

$$\mu_e = \frac{Velocity\ of\ light\ in\ air}{Maximum\ normal\ velocity\ of\ E\ wave}$$

The principal refractive index of extra-ordinary wave in positive uniaxial crystals is expressed as

$$\mu_e = \frac{Velocity\ of\ light\ in\ air}{Minimum\ normal\ velocity\ of\ E\ wave}$$

Huygens's construction of O and E wave front: In all the following cases AB represents the trace of an incident plane wave front perpendicular to the plane of the paper. It is incident on the refracting face XX^l of a doubly refracting negative crystal (calcite).

Case 1: *Optic axis lying in the plane of incidence and inclined to the refracting surface.*

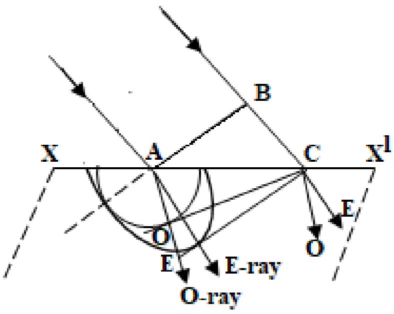


Fig 1

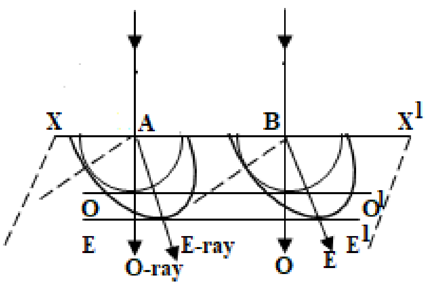


Fig 2

Oblique incidence:-

AB is the incident wave front which meet obliquely at the surface XX^l of the crystal (Fig 1) first at the point A and makes it the centre of ordinary and extra-ordinary wavelets. During the time $t = \frac{BC}{V_a}$ in which the disturbance from B reaches, the ordinary spherical wavelet has travelled a distance equal to

$$t \times V_0 = \frac{BC}{V_a}.V_0 = \frac{BC}{V_a/V_0} = \frac{BC}{\mu_0}$$

Where $V_a$the velocity of light in air , V_0 is the velocity of light of O-ray and μ_0 is the R.I of O-ray. To find the position of the O-wave front in the crystal draw a sphere with A as the centre and $\frac{BC}{V_a}$ as the radius. From C draw a tangent plane CO touching the sphere at O. A plane passing through CO represents the position of the O-wave front.

Similarly for extra-ordinary ray the distance travelled is

$$t \times V_e = \frac{BC}{V_a}.V_e = \frac{BC}{V_a/V_e} = \frac{BC}{\mu_e}$$

Where $V_a$the velocity of light in air, V_e is the velocity of light of E-ray and μ_e is the R.I of E-ray. The tangent plane CE touching the ellipsoid at E represents the position of the E-wave front. It is clear that O-ray and E-ray travel along different directions with different velocities.

Normal incidence:- As in the figure 2 draw tangent planes OO^l and EE^l respectively to the spherical and ellipsoidal surface originating from the centers A and B where the incident waves strike the crystal simultaneously. The planes though OO^l and EE^l represent the O and E wave fronts in the crystal. O and E rays travel in different directions with different velocities.

Case 2: *Optic axis lying in the plane of incidence and perpendicular to the refracting surface.*

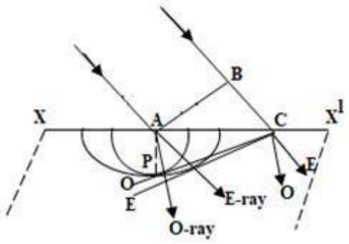


Fig 1

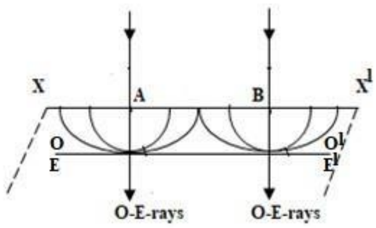


Fig 2

Oblique incidence:-

In this case the two wave fronts of ordinary and extra-ordinary rays touch each other at P (Fig 1) the line joining AP represents the direction of the optic axis. Proceeding in the same manner as in case 1 above, the plane passing through CO and CE respectively represent the wave front of the O and E waves. The O and E rays travel with different velocities in different directions.

Normal incidence:-

O and E waves coincide at all instant (Fig 2) and travel in the same direction with the same velocity. Hence there is no double refraction in this case.

Case 3: *Optic axis lying in the plane of incidence and parallel to the refracting face.*

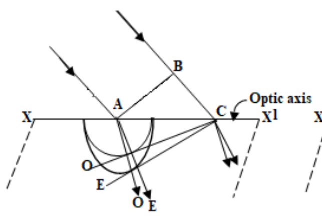


Fig 1

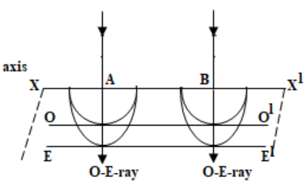


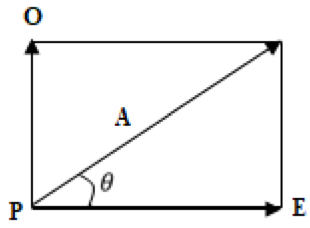
Fig 2

Oblique incidence:- In this case the optic axis lies along the line XX^1 as in Fig. The two secondary surfaces, the sphere and the ellipsoid touch on this line (Fig 1). The position of the O and E wave fronts CO and CE are drawn on exactly the same consideration as in case 1. The O and E refracted wave fronts travel with different velocities along different directions.

Normal incidence:-

In this case OO^1 and EE^1 are the refracted wave fronts at the same instant of time (Fig 2), they are parallel to each other and travel in the same direction but with different velocities, thus producing path difference between the O and E waves on emergence, but there is no separation into two beams. This property is made use in the construction of quarter wave and half wave plates.

Theory of plane, elliptically and circularly polarized light:



Suppose the amplitude of the incident plane polarized light in the crystal is A and it makes an angle θ with the optic axis (Fig).Therefore the amplitude of the ordinary ray vibrating along PO is $A\sin\theta$and the amplitude of the extraordinary ray vibrating along PE is $A\cos\theta$. Since a phase difference δ is introduces between the two rays, after coming out of the crystal can be represented in terms of two simple harmonic motions, at right angles to each other and having a phase difference.

For extraordinary ray,

$$x = A\cos\theta\,.\sin(\omega t + \delta)$$

For ordinary ray,

$$y = A\sin\theta\,.\sin\omega t$$

Taking $A\cos\theta = a\ \ and\ \ A\sin\theta = b$, we have,

$$x = a\sin(\omega t + \delta)\ldots\ldots\ldots\ldots\ldots(1)$$

$$y = \ b\sin\omega t\ldots\ldots\ldots(2)$$

From equation (2),

$$\frac{y}{b} = \sin\omega t$$

$$\cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

From equation (1), $\frac{x}{a} = \sin\omega t.\cos\delta + \cos\omega t.\sin\delta$

Substituting the values of $sin\omega t\ \ and\ \ cos\omega t$, we get,

$$\frac{x}{a} = \frac{y}{b}.cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\ sin\delta$$

$$\frac{x}{a} - \frac{y}{b}.cos\delta = \sqrt{1 - \frac{y^2}{b^2}}\ sin\delta$$

On squaring, we have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} cos^2\delta - \frac{2xy}{ab} . cos\delta = (1 - \frac{y^2}{b^2})\ sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} cos^2\delta - \frac{2xy}{ab} . cos\delta = sin^2\delta - \frac{y^2}{b^2} sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} cos^2\delta + - \frac{y^2}{b^2} sin^2\delta - \frac{2xy}{ab} . cos\delta = sin^2\delta$$

$$\boldsymbol{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} . cos\delta = sin^2\delta} \ldots \ldots \ldots \ldots (\mathbf{3})$$

This is general equation of an ellipse.

Special cases: (1) When $\delta = 0$, equation (3) becomes,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$(\frac{x}{a} - \frac{y}{b})^2 = 0$$

$$\frac{x}{a} = \frac{y}{b}$$

$$y = \frac{b}{a} . x$$

This is the equation of a straight line. Therefore, the emergent light will be plane polarized.

(2) When $\delta = \pi/2$, equation (3) becomes, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

This represents the equation of symmetrical ellipse. The emergent light in this case will be elliptically polarized provided $a \neq b$.

(3) When $\delta = \pi/2$, and $a = b$, equation (3) becomes,

$$x^2 + y^2 = a^2$$

This represents the equation of circle of radius a. The emergent light will be circularly polarized. Here the vibration of the incident plane polarized on the crystal make an angle of 45^0 with the direction of the optic axis.

Quarter wave plate: It is a uniaxial doubly refracting crystal plate, of suitable thickness, cut with its optic axis parallel to the refracting face and can introduce a phase difference of $\pi/2$ (or path difference of $\lambda/4$) between the O and E-rays.

When a beam of plane polarized monochromatic light of wavelength λ is incident normally on a crystal of this type it decomposes into O and E components which travel in the same direction but with different velocities. If 't' be the thickness of the plate then the distance travelled in the crystal is equivalent to distances $\mu_0 t\ and\ \mu_e t$ in air for the O and E rays respectively. Therefore the resultant path difference between the two components will be

$$P.D = t(\mu_0 - \mu_e)$$

We know that the relation between phase difference and path difference is given by

$$Phase\ difference = \frac{2\pi}{\lambda} . Path\ difference$$

To introduce a phase difference of $\frac{\pi}{2}$ between O and E rays, we have,

$$\frac{\pi}{2} = \frac{2\pi}{\lambda} . t(\mu_0 - \mu_e)$$

$$t = \frac{\lambda}{4.(\mu_0 - \mu_e)}$$

This is the expression for the thickness of the quarter wave plate. Similarly for +ve crystal (Quartz), the expression for thickness of the quarter wave plate is

$$t = \frac{\lambda}{4.(\mu_e - \mu_0)}$$

When plane polarized light is incident on a quarter wave plate, the emergent light is in general elliptically polarized. If plane of polarization of the incident beam makes an angle of 45^0 with the optic axis then the emergent beam is circularly polarized light.

Thus quarter wave plate is the simplest device for producing and detecting circularly polarized light. In conjunction with a Nicol prism it is used for analysing all kinds of polarized light.

Half wave plate: It is a uniaxial doubly refracting crystal plate, of suitable thickness, cut with its optic axis parallel to the refracting face and can introduce a phase difference of π (or path difference of $\pi/2$) between the O and E-rays.

When a beam of plane polarized monochromatic light of wavelength λ is incident normally on a crystal of this type it decomposes into O and E components which travel in the same direction but with different velocities. If 't' be the thickness of the plate then the distance travelled in the crystal is equivalent to distances $\mu_0 t \ and \ \mu_e t$ in air for the O and E rays respectively. Therefore the resultant path difference between the two components will be

$$P.D = t(\mu_0 - \mu_e)$$

We know that the relation between phase difference and path difference is given by

$$Phase\ difference = \frac{2\pi}{\lambda}.Path\ difference$$

To introduce a phase difference of π between O and E rays, we have,

$$\pi = \frac{2\pi}{\lambda} . t(\mu_0 - \mu_e)$$

$$t = \frac{\lambda}{2 (\mu_0 - \mu_e)}$$

This is the expression for the thickness of the quarter wave plate. Similarly for +ve crystal (Quartz), the expression for thickness of the half wave plate is

$$t = \frac{\lambda}{2 (\mu_e - \mu_0)}$$

When a plane polarised light is incident upon a half wave plate, the transmitted light is also plane-polarised. If the plane of polarisation of the incident light makes an angle θ with the direction of the optic axis, the plane of polarisation of the emergent beam makes an angle $-\theta$ with the same direction that is the plane has effectively been rotated through an angle 2θ.

A half wave plate finds its application in the construction of Laurent's half shade device used in a polarimeter.

Production of Plane, Elliptical and Circularly polarized light:

Plane polarized light:

A beam of monochromatic light is passed through a Nicol prism, while passing through the prism, the beam split up into O-ray and E-ray. The O-ray is totally internally reflected back at the Canada balsam layer, while the E-ray passes through the N-prism. The emergent beam is plane polarized.

Circularly polarized light:

Circularly polarized light is the resultant of two waves of *equal amplitude,* vibrating at right angles to each other and having a phase difference of $\pi/2$.

A parallel beam of monochromatic light is allowed to fall on a Nicol prism the beam after passing through the N-prism is plane polarized now place another N-prism at some distance from the first one in the crossed position, that is the field of view will be dark as viewed by the eye in this position. A quarter wave plate is introduced between N-prisms and it is rotated until the field of view is dark, then from this position quarter wave plate is rotated through 45^0. In this case, the vibrations of the plane polarized light falling on the quarter wave plate makes an angle 45^0 with the direction of the optic axis of the quarter wave plate. The light which is emerging out from this *quarter wave plate* is circularly polarized.

Elliptically polarized light: Elliptically polarized light is the resultant of two waves of *unequal amplitude* vibrating at right angles to each other and having a phase difference of $\pi/2$.

A parallel beam of light is allowed to fall on an N-prism the beam after passing through the N-prism is plane polarized. Now place another N-prism at some distance from the first one in the crossed position so that field of view is dark as viewed by the eye in this position. A quarter wave plate is introduced between the two N-prisms. The plane polarized light from the first N-prism falls normally on the quarter wave plate. The light which is emerging out of QWP is elliptically polarized (the precaution should be taken that light should not incident at an angle of 45^0 with optic axis).

Detection of plane, elliptical and circularly polarized light:

Plane polarized light: The beam is allowed to fall on a N-prism. On rotation of the N-prism, if light extinguishes completely twice in each rotation, then the beam is plane polarized.

Circularly polarized light:

The beam is allowed to fall on an N-prism. The intensity of the beam remains uniform when the N-prism is rotated. The beam in this case is either *circularly* polarized or *unpolarised*.

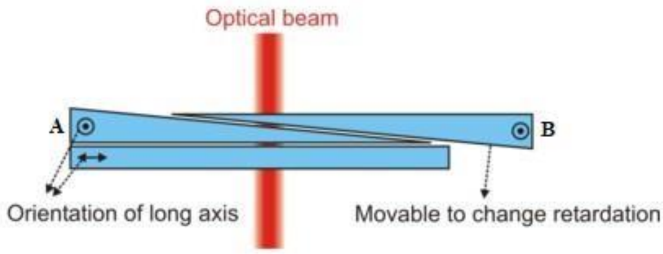
To distinguish between the two, the beam is allowed to fall on a quarter wave plate and then on an N-prism. If the beam is circularly polarised after passing through the quarter wave plate, the O-ray and E-rays will undergo a further phase difference of $\pi/2$. The beam after passing through the quarter wave plate becomes *plane polarised*. This beam is passed through a rotating N-prism. If the light extinguished completely twice in each rotation then the incident beam is *circularly polarised* light otherwise it is unpolarised.

Elliptically polarised light:

When elliptically polarised light is passed through a rotating N-prism, the intensity varies from maximum to minimum. This behaviour is same even for partially polarised light (combination of polarized and unpolarised).

To distinguish between the two, the elliptically polarised light is passed through quarter wave plate and then on N-prism. If the beam is elliptically polarised the O-ray and E-ray will undergo a further phase difference of $\pi/2$. The beam after passing through the quarter wave plate becomes plane polarised. If this beam extinguishes completely twice in each rotation, then the incident beam is elliptically polarised otherwise it is partially polarised.

Babinet's compensator: It is an optical device employed in the production and analysis of elliptically polarised light.



A quarter wave plate or a half wave plate produces only a fixed path difference between the ordinary and the extra-ordinary rays and can be used only for light of particular wavelengths. For different wavelengths different quarter or half wave plates are to be used. To avoid this difficulty, Babinet designed a compensator by means of which a desired path difference can be introduced.

It consists of two wedge shaped sections A & B of quartz, they are mounted in such a way that it forms a rectangular block. The crystal A is fixed and B can slide along the surface of A with the help of micrometer screw (figure), by this the thickness of the optical path can be varied for desired value. Thus Babinets compensator will introduce any desired path difference and it can be used for light of any wavelength.

Optical activity: When a plane polarised light is made to pass through a certain substances the plane of polarization of the emergent beam has been rotated through a certain angle. This phenomenon is known as optical activity and the substances which rotate the plane of polarization are said to be optically active.

There are two types of optically active substances. Substances which produces clock wise rotation are known as dextrorotatory (right-handed) substances and the substances which rotate the plane of polarization in the anti-clock wise direction are called laevorotatory (left handed) substances.

Examples: Aqueous solutions of various kinds of sugars and of tartaric acid, turpentine oil. Quartz etc.

Fresnel's explanation of optical rotation: (*Fresnel's theory*)

Fresnel's explanation of optical rotation is based upon the assumption that when plane polarized light is allowed to pass through the substance, it decomposes into two circularly polarized vibrations rotating in opposite direction with the same frequency.

In an optically inactive substance, as these two vibrations travels with the same velocity, the plane of emergence lies in the plane of the incidence hence there is no optical rotation (Fig 1). In dextrorotatory substance, right handed motion travels faster than the left handed (Fig 2) while in a laevorotatory substances left-handed motion travels faster than the right handed (Fig 3). As a result of this the rotation of the plane of polarization is to the right or left according as the right handed or the left handed component is faster and is equal to half the phase difference on emergence between the two circular vibration as shown in the figure. These results may be arrived easily by means of the equations of the vibration.

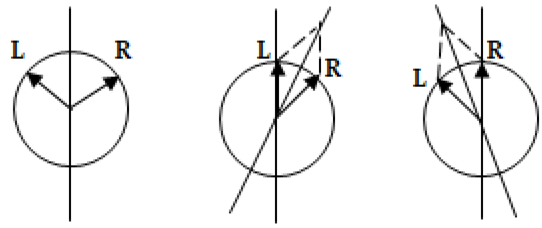


Fig 1 Fig 2 Fig 3

Optically inactive substance:

For clockwise circular vibration

$$x_1 = acos\omega t; \qquad y_1 = asin\omega t$$

For anti-clock wise circular vibrations

$$x_2 = -\ acos\omega t; \qquad y_2 = asin\omega t$$

Therefore the resultant vibrations are

$$X = x_1 + x_2 = \ acos\omega t - \ acos\omega t = 0$$

$$and\ Y = y_1 + y_2 = asin \quad t + asin\omega t = 2asin\omega t$$

Thus the resultant vibration has amplitude 2a and is plane polarized. The plane of vibration is along the original direction. That is, along the plane of incidence hence there is no optical rotation (Fig 1).

Optically active substance (dextrorotatory):

For clockwise circular vibration

$$x_1 = acos\omega t; \qquad y_1 = asin\omega t$$

For anti-clock wise circular vibrations

$$x_2 = -\ acos(\omega t + \delta); \qquad y_2 = asin(\omega t + \delta)$$

Therefore the resultant vibrations are

$$X = x_1 + x_2 = \ acos\omega t - \ acos(\omega t + \delta)$$

$$= -2asin\left(\frac{\omega t + \omega t + \delta}{2}\right).sin\left(\frac{\omega t - \omega t - \delta}{2}\right)$$

$$= -2asin\left(\omega t + \frac{\delta}{2}\right).sin\left(\frac{-\delta}{2}\right)$$

$$= 2asin\left(\omega t + \frac{\delta}{2}\right).sin\left(\frac{\delta}{2}\right) \ldots\ldots\ldots..(1)$$

$$and\ Y = y_1 + y_2 = asin\omega t + asin(\omega t + \delta)$$

$$= 2asin\left(\frac{\omega t + \omega t + \delta}{2}\right).cos\left(\frac{\omega t - \omega t - \delta}{2}\right)$$

$$= 2asin\left(\omega t + \frac{\delta}{2}\right).cos\left(\frac{-\delta}{2}\right)$$

$$= 2asin\left(\omega t + \frac{\delta}{2}\right).cos\left(\frac{\delta}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

From equations (1) and (2) it is clear that the plane of polarization of the emergent beam has been rotated in the clock wise direction (right hand side). The angle through which the plane has been rotated can be calculated by dividing equations (1) by (2)

$$\tan\theta = \frac{X}{Y} = \frac{\sin\frac{\delta}{2}}{\cos\frac{\delta}{2}} = \tan\frac{\delta}{2}$$

$$\therefore \quad \boldsymbol{\theta = \frac{\delta}{2}}$$

That is the plane is rotated through half the phase difference between two circular vibrations. One can prove the same relation for laevorotatory substance.

Specific rotation:

Liquid containing an optically active substance rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon,

1. The length of the solution
2. The concentration of the solution
3. The wavelength of light and
4. The temperature.

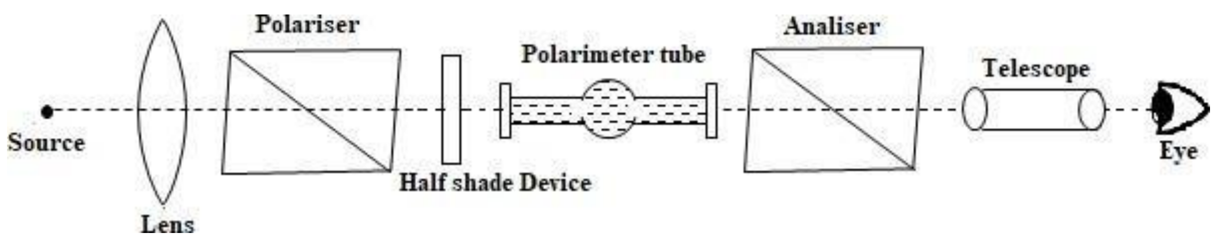
$$\theta \propto l.C$$

$$\theta = {}_tS^{\lambda}.\, l.C$$

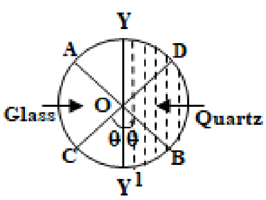
$$S_t^{\lambda} = \frac{\theta}{l.C}$$

Hence the specific rotation of the solution is defined as "The amount of rotation produced by a solution of unit length having unit concentration at constant temperature and for light of given wavelength".

Laurent's half shade polarimeter:



It consists of two Nicol prisms polariser and analyser. Behind polariser there is half shade device of quartz which covers one half of the field of view while the other half is a glass plate. The glass plate absorbs same amount of light as the quartz plate. Polarimeter tube is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution there should be no air bubble in the path of light. If any air bubble will appear it should be brought at the upper portion of the wide bore of the tube.



Light from a monochromatic source is incident on the converging lens. After passing through polariser the beam is plane polarised. One half of the beam passes through the quartz plate and the other half passes through the glass plate. Suppose the plane of vibration of the plane polarised light incident on the half shade plate is along AB. Here AB makes an angle θ with YY^{l}. On passing through the quartz plate the beam split up into ordinary and extraordinary components and on emergence a phase difference of π is introduced between them. The vibrations of the beam emerging out of quartz will beam along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyser has its principal plane along YY^{l}, the amplitude of light incident on the analyser both the halves will be equal. Therefore the field of view will be equally bright.

If the analyser is rotated to the right of YY^{l}, then the right half will be brighter as compared to the left half. On the other hand if the analyser is rotated to the left of YY^{l} the left half is brighter as compared to the right half.

Therefore to find the specific rotation of an optically active substance, the analyser is set in the position for equal brightness of field of view first without the solution in the tube. The readings of the Vernier are noted. When a tube containing the solution of known concentration is placed, on introduction of the tube the field of view is not equally bright as set earlier. The analyser is rotated in the clockwise direction until the field of view is equally bright again. The new position of the Vernier reading is noted. The difference in the two readings gives angle through which the plane of polarisation of the incident beam has been rotated by the sugar solution. Experiment is repeated for various concentrations, the corresponding angles of rotation are determined. A graph is platted between concentration C and the angle of rotation θ. The graph is a straight line. Then from the relation, $S = \frac{\theta}{l\,C}$ the specific rotation of the optically active substance is calculated.

Problems

1. Plane polarized light passes through a quartz plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam (i) will be plane polarized and (ii) will be circularly polarized light. Given $\mu_e = 1.553, \mu_0 = 1.544 \; and \; \lambda = 5000$ Å.

Solution:

$$For\ plane\ polarised\ light;\ \ t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{5000 \times 10^{-10}}{2 \times (1.553 - 1.544)} = 2.778 \times 10^{-5}\ m$$

$$For\ circularly\ polarised\ light;\ \ t = \frac{\lambda}{4(\mu_e - \mu_0)} = \frac{5000 \times 10^{-10}}{4 \times (1.553 - 1.544)} = 1.388 \times 10^{-5}\ m$$

2. Calculate the least thickness of a calcite plate which convert plane polarized into circularly polarized light, if the refractive indices of calcite for sodium light are respectively 1.658 and 1.486 for the ordinary ray and the extraordinary ray. Given $\lambda = 589.3\ nm$

Solution: $\mu_0 = 1.658, \mu_e = 1.486 \; and \; \lambda = 589.3\ nm.$

$$For\ circularly\ polarised\ light;\ \ t = \frac{\lambda}{4(\mu_0 - \mu_e)} = \frac{589.3 \times 10^{-9}}{4 \times (1.658 - 1.486)} = 8.565 \times 10^{-7}\ m$$

3. A crystal plate produces an optical path difference of $1.5 \times 10^{-7}\ m$ between the ordinary and extraordinary vibrations calculate the thickness of the plate. Given $\mu_e = 1.554, \mu_0 = 1.544$

Solution: Expression for path difference is given by

$$P.D = t(\mu_e - \mu_0)$$

$$t = \frac{P.D}{(\mu_e - \mu_0)} = \frac{1.5 \times 10^{-7}}{(1.554 - 1.544)} = 1.5 \times 10^{-5}\, m$$

4. Calculate the rotation of the plane of polarisation in a substance of unit thickness for a light of wavelength $5890 \times 10^{-10}\, m$. The difference between the refractive indices for right and left circularly polarized light in the substance is 7.62×10^{-8}.

Solution: The relation between the phase difference and the path difference is given by

$$Phase\ diffeerence = \frac{2\pi}{\lambda}.Path\ difference$$

$$Phase\ diffeerence = \phi = \frac{2\pi}{\lambda}.t(\mu_L - \mu_R) = \frac{2 \times 3.14 \times 7.62 \times 10^{-8} \times 1}{5890 \times 10^{-10}} = 0.8125\, rad$$

$$\phi = 0.8125\ \times \frac{180}{\pi} = 46.57^0$$

5. The rotation of the plane of polarisation is 40^0 in a substance of thickness 2 m. If the difference between the refractive indices for left and right circularly polarized light in the substance is 3.2738×10^{-8}, calculate the wavelength of light used.

Solution:

$$Phase\ diffeerence = \frac{2\pi}{\lambda}.Path\ difference$$

$$\phi = \frac{2\pi}{\lambda}.t(\mu_L - \mu_R)$$

$$\lambda = \frac{2\pi}{\phi}.t(\mu_L - \mu_R) = \frac{2 \times 3.14 \times 3.2738 \times 10^{-8} \times 2 \times 180}{40 \times \pi} = 5.8928 \times 10^{-7}\, m$$

6. Sugar solution of concentration 100 kgm^{-3} is kept in a polarimeter tube of length 0.22 m. If the specific rotation of sugar is 0.75^0 $kg^{-1}m^2$, calculate the rotation of the plane of polarization.

Solution: Given:- $C = 100\ kgm^{-3}, l = 0.22\ m, S = 0.750\ kg^{-1}m^2, \theta = ?$

$$S = \frac{\theta}{lC}$$

$$\theta = S.l.C = 0.75 \times 0.22 \times 100 = 16.5^0$$

7. A 0.2 m long polarimeter tube containing a certain solution of concentration 20% produces an optical rotation of 24^0. Find the specific rotation of the solution.

Solution: Given:- $l = 0.2\, m, C = 20\% = \frac{20}{100}\, gcm^{-3} = 0.2 \times 10^{-3} \times 10^6\ kgm^{-3}, \theta = 24^0, S = ?$

$$S = \frac{\theta}{lC} = \frac{24}{0.2 \times 0.2 \times 10^3} = 0.6^0$$

8. Determine the concentration of a solution of length 0.25 m which produces an optical rotation of 30^0. The specific rotation of the solution is 0.0209 rad m^2 kg^{-1}.

Solution: Given:- $l = 0.25\, m,\ \theta = 30^0,\ S = 0.0209\ \text{rad m2 kg}^{-1} = 1.198^0,\ C = ?$

$$S = \frac{\theta}{lC}$$

$$C = \frac{\theta}{lS} = \frac{30}{0.25 \times 1.198^0} = 100.166\ kg\ m^{-3}$$

9. A 0.2 m long tube containing $48 \times 10^{-6}\ m^3$ of sugar solution produces an optical rotation of 11^0 when placed in a saccharimeter. If the specific rotation of sugar solution is 0.66^0. Calculate the quantity of sugar contained in the tube in the form of a solution.

Solution: Given:- $l = 0.2\ m,\ \theta = 11^0,\ V = 48 \times 10^{-6}\ m^3, S = 0.66^0,\ M = ?$

$$S = \frac{\theta}{lC}$$

$$C = \frac{\theta}{lS} = \frac{11}{0.2 \times 0.66^0} = 83.33\ kg\ m^{-3}$$

$$\therefore\ M = C \times V = 83.33 \times 48 \times 10^{-6} = 0.003999\ kg$$

$$M = 4\ g$$

10. 0.2 m length of a certain optically active solution causes right handed rotation of 40^0 and 0.3 m of another solution causes left handed rotation of 24^0. What will be the optical rotation produced by 0.3 m length of the mixture of the above solutions in volume ratio 1:2. It is given that the solution do not react chemically.

Solution: As the length of the mixture is 0.3 m and the solutions are in the volume ratio 1:2, we may assume that **0.1** m length of the first solution and **0.2** m length is of the second solution.

$$The\ optical\ rotation\ produced\ by\ the\ first\ solution = \frac{0.1 \times 40}{0.2} = 20^0\ Right\ handed(+ve)$$

$$The\ optical\ rotation\ produced\ by\ the\ second\ solution = \frac{0.2 \times 24}{0.3} = 16^0\ Left\ handed(-ve)$$

$$\therefore Total\ optical\ rotation = 20^0 - 16^0 = 4^0$$

$$\therefore\ The\ resu\quad tant\ optical\ rotation\ is\ 4^0\ Right\ handed(since\ the\ value\ is\ +ve)$$

Note:

First solution	Second solution
$0.2 \longrightarrow 40^0$	$0.3 \longrightarrow 24^0$
$0.1 \longrightarrow ?$	$0.2 \longrightarrow ?$
$\frac{0.1 \times 40}{0.2} = 20^0$	$\frac{0.2 \times 24}{0.3} = 16^0$

Unit-4 LASER PHYSICS

Introduction

The word LASER is an acronym for *"Light Amplification by Stimulated Emission of Radiation"*. It is a powerful monochromatic light source of collimated beam in which the light waves are highly coherent. The laser light has many superior features compared to conventional light source. Einstein introduced this concept in 1917. Dr. T.H. Maiman demonstrated the first laser namely the ruby laser in the year 1960.

Characteristics of LASER :

Laser differs from the ordinary light with respect to some properties. They are

- Monochromaticity
- Directionality
- Coherence
- Intensity

1. Monochromaticity :

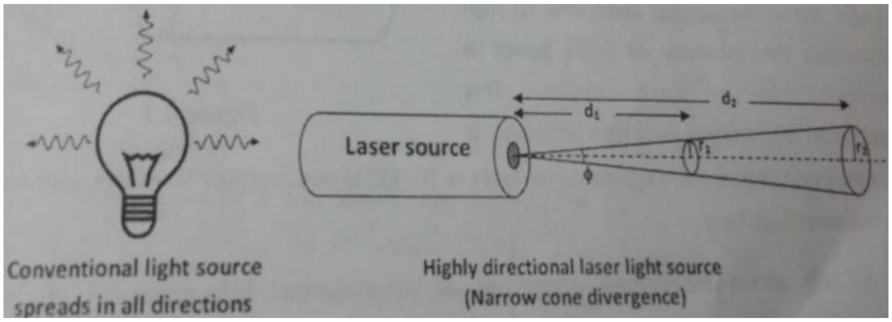
Laser beam is highly monochromatic. It emits single wavelength (one colour) because atoms or molecules are transition between two energy states. Hence it possesses good spectra since range of laser beam wavelength ($\Delta\lambda$) is very narrow. But ordinary light emits combination of wide range of wavelength (colors) because atoms or molecules are transition from several number of excited states to ground state, so it emits different energies, therefore it is polychromatic.

2. Directionality or Divergence :

The light ray coming ordinary light source travels in all directions, but laser light travels in single direction. For example, the light emitted from torch spreads 1km distance, But the laser light spreads a few centimeters distance even it travels longer distance. The ordinary source emits light in all directions and its angular spread is 1 metre/metre. But the laser is highly directional and non-divergent and its angular spread is 1mm/metre.

The angular spread (ϕ) or divergence is given by,

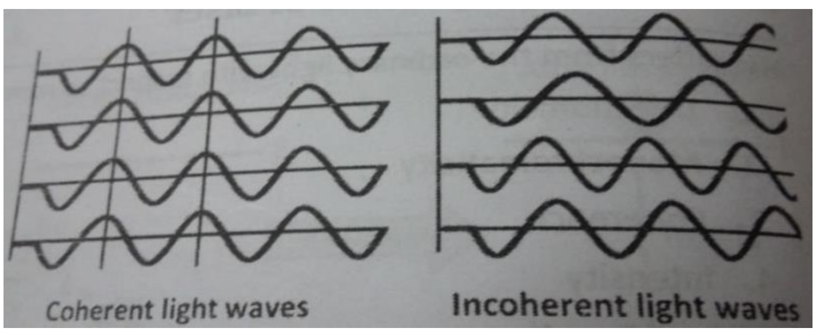
$$\Phi = \frac{r_2 - r_1}{D_2 - D_1} \quad degree$$



where D1, D2 are any two distances from the laser source emitted and r1, r2 are the radii of the beam spots at a distance D1 and D2, respectively.

3. Coherence

A predictable correlation of the amplitude and phase at any one point with other point is called coherence. The light from a source consists of wave pattern. These wave patterns when identical in phase and direction are called coherent. Laser has a high degree of coherence than the ordinary sources. The coherence of laser emission results in an extremely high power of 5×10^6 watt/m^2. A laser beam can be focused to a very small area of about 0.7 μm diameter.



4. Intensity

Laser light is highly intense than the ordinary light. This is because of coherence and directionality of laser. The ordinary light spreads in all directions, so the intensity reaching the target is very less. But in the case of laser, due to high directionality the intensity of laser beam is concentrated in a small region. This concentration of energy gives a high intensity. Since in Laser many number of photons are in phase with each other, the amplitude of the resulting wave becomes na and hence the intensity of laser is proportional to n^2a^2. It is estimated that light from a typical 1mW laser is 10,000 times brighter than the light from the sun at the earth's surface.

Ordinary light versus LASER :

Ordinary light	LASER
Ordinary light are Polychromatic since it consists of radiations of several wavelengths.	Laser light is monochromatic since it consists of only single wavelength.
Ordinary light is divergent because it spreads in all directions	Laser light is non-divergent because it travels in single direction
Intensity of ordinary light is lesser because concentration of photons is lesser	Intensity of Laser light is higher because concentration of photons is higher
Ordinary light is not Coherent	Laser light is highly Coherent
Due to less intensity brightness of ordinary light is low	Due to high intensity brightness of Laser light is high
Examples: Sunlight	Example: Laser light

Ground state and Excited states:

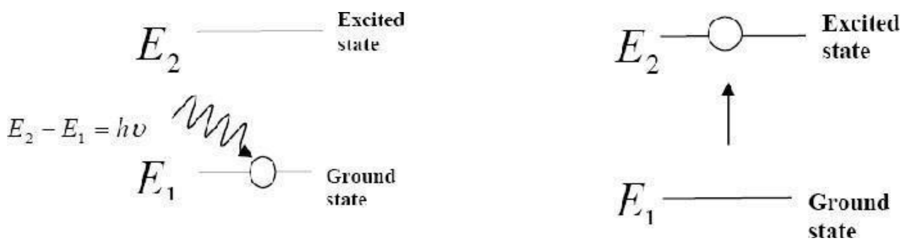
The lowest energy level for an individual atom occurs when its electrons are all in the nearest possible orbits to its nucleus ,this energy level is called the "**Ground state**".

When one or more of an atom's electrons have absorbed energy, they can move to outer orbits, and the atom is then referred to as being excited, and that energy level is called as **"Excited state."** Excited states are generally not stable; as electrons drop from higher-energy to lower-energy levels, they emit the extra energy as light.

Interaction of Radiation with matter

1. Induced Absorption or Stimulated Absorption :

An atom is in the ground state with energy E_1 absorbs a photon of energy hν and goes to the excited state with energy E_2 as shown in Fig. This transition is known as stimulated absorption or induced absorption or simply absorption. Here the energy difference is given as $(E_2 - E_1) = h\nu$.



If there are many number of atoms in the ground state then each atom will absorb the energy from the incident photon and goes to the excited state then,

The rate of absorption (R12) is proportional to,

R12 $\propto$ Energy density of incident radiation ($\rho\nu$)

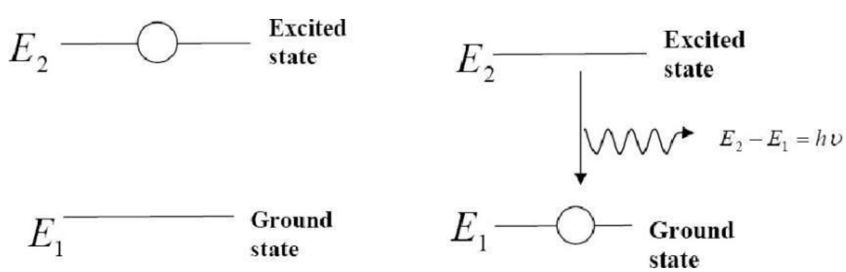
$\propto$ No. of atoms in the ground state (N1)

$$R_{12} = B_{12} \rho\nu N_1$$

Where, B12 is a constant which gives the probability of absorption transition per unit time.

2. Spontaneous emission

The natural tendency of an atom is to seek out the lowest energy configuration. The excited atoms do not stay in the excited state for longer time but tend to return to the lower state by giving up the excesses energy $h\nu$ as shown in fig. The atom in the excited state E2 returns to the ground state E1 by emitting a photon of energy $h\nu$ without any external energy. Such emission of radiation not initiated by any external influence is called spontaneous emission. This emission is uncontrollable.



The rate of spontaneous emission R21 (Sp) is proportional to number of atoms or molecules present in the excited state.

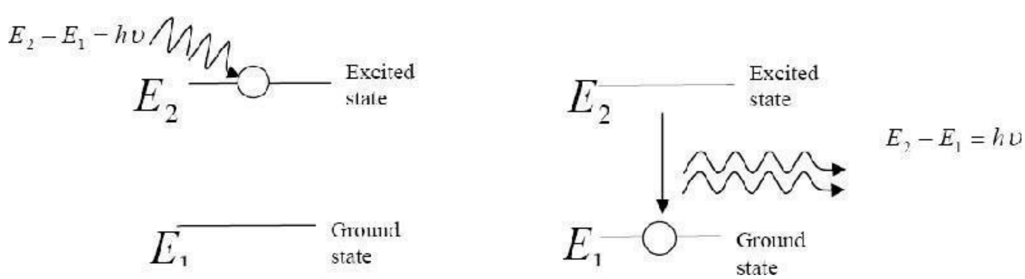
i.e R21 (Sp) $\propto$ N2

R21 (Sp) = A21 N2

where A21 is a constant which gives the probability of spontaneous emission transitions per unit time.

3. Stimulated emission

The atom in the excited state E2 as shown in fig. A photon of energy $h\nu$ can stimulate the atom to move to its ground state. During this process the atom emits an additional photon whose energy is also $h\nu$. As the emission is stimulated by external photon, this process is known as stimulated emission.



The rate of stimulated emission R21 (St) is proportional to,

R21 $\propto$ Energy density of incident radiation (ρ_ν)

$\propto$ No. of atoms in the excited state (N_2)

i.e R21 (St) $\propto$ $\rho_\nu N_2$

R21 (St) = $B_{21} \rho_\nu N_2$

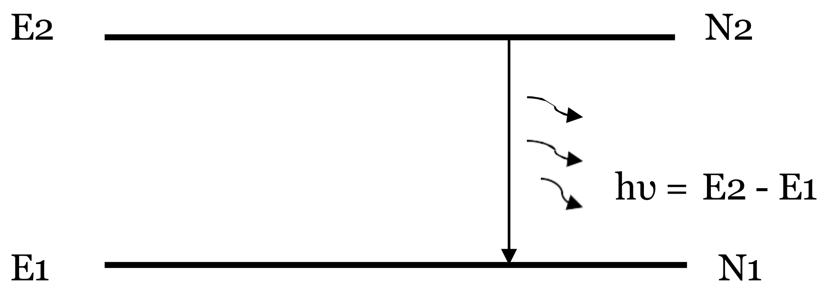
where B21 is a constant which gives the probability of stimulated emission transitions per unit time.

Einstein's A and B Coefficient:

(Relation between Einstein's coefficient and Energy density of radiation)

Einstein's theory of absorption and emission of light by an atom is based on Planck's theory of radiation.

Consider two energy levels E1 and E2 having number of atoms N1 and N2 respectively, as shown in figure.



According to Einstein's theory,

Rate of Stimulated absorption is, $\mathbf{R_{12} = B_{12}} E_\nu \mathbf{N_1}$

Rate of Spontaneous Emission is, $\mathbf{R_{21(Sp)} = A_{21} N_2}$

Rate of Stimulated Emission is, $\mathbf{R_{21(St)} = B_{21}} E_\nu \mathbf{N_2}$

Where, A and B represents the Spontaneous and Stimulated process respectively. E_υ is the energy density of radiation.

At thermal equilibrium,

The rate of absorption = The rate of emission

$B_{12} E_\nu N_1 = A_{21} N_2 + B_{21} E_\nu N_2$---------------- (1)

$E_\nu [B_{12} N_1 - B_{21} N_2] = A_{21} N_2$

$$E_\upsilon = \frac{A_{21} N_2}{[B_{12} N_1 - B_{21} N_2]}$$

$$E_\upsilon = \frac{A_{21}}{[B_{12} \frac{N_1}{N_2} - B_{21}]} \text{------------------------------ (2)}$$

Under thermal equilibrium, the population of energy levels obeys the Boltzmann's distribution law.

We know from Boltzmann distribution law,

$$N_1 = N_0\, e^{\frac{-E_1}{KT}}$$

$$N_2 = N_0\, e^{\frac{-E_2}{KT}}$$

Where, K is the Boltzmann constant,

T is the absolute temperature and

N_o is the number of atoms at absolute zero at equilibrium,

we can write the ratio of population as follows,

$$\frac{N_1}{N_2} = e^{\frac{E_2 - E_1}{KT}}$$

since $E_2 - E_1 = h\nu$, we have

$$\therefore \frac{N_1}{N_2} = e^{\frac{h\boldsymbol{\nu}}{KT}} \text{------------------------------------} (3)$$

Substitute eq(3) in eq(2), we get

$$E_\upsilon = \frac{A_{21}}{[B_{12} e^{\frac{hu}{KT}} - B_{21}]}$$

Divide both numerator and denominator by B_{12}, we get

$$E_\upsilon = \frac{\frac{A_{21}}{B_{12}}}{[e^{\frac{hu}{KT}} - \frac{B_{21}}{B_{12}}]} \quad ---------(4)$$

According to Maxwell-Boltzman statistics

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h u^3}{c^3} \quad \text{and} \quad \frac{B_{21}}{B_{12}} = 1$$

Equation (4) becomes,

$$E_\upsilon = \frac{8\pi h \upsilon^3}{c^3\, [e^{\frac{hu}{KT}} - 1]}$$

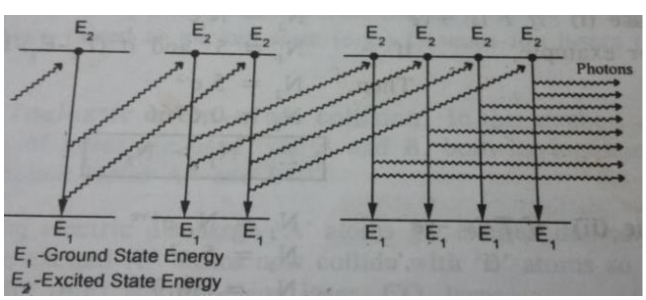
This is the relation between Einstein's coefficient and Energy density of radiations(E_υ).

Difference between Stimulated and Spontaneous Emission :

Sl.No	Stimulated Emission	Spontaneous Emission
1.	An atom in the excited state is induced to return to ground state, thereby resulting in two photons of same frequency and energy is called stimulated emission.	The atom in the excited state returns to ground state thereby emitting a photon, without any external inducement is called spontaneous emission.
2.	The emitted photons move in same direction and are highly directional.	The emitted photons move in all directions and are random.
3.	The radiation is high intense, monochromatic and coherent.	The radiationis less intense and is incoherent.
4.	The photons are in phase.	The photons are not in phase.
5.	The rate of transition is given by R21(St) = B21 ρν N2	The rate of transition is given by R21(Sp) = A21N2

Light amplification :

Let us consider many numbers of atoms in the excited state. We know the photons emitted during stimulated emission have same frequency, energy and are in phase as the incident photon. Thus results in 2 photons of similar properties. These two photons induce stimulated emission of 2 atoms in excited state thereby resulting in 4 photons. These 4 photons induce 4 more atoms and give rise to 8 photons etc., as shown in Fig.



Principle: Due to stimulated emission the photons multiply in each step giving rise to an intense beam of photons that are coherent and moving in the same direction. Hence the Light is Amplified by Stimulated Emission of Radiation, termed as LASER.

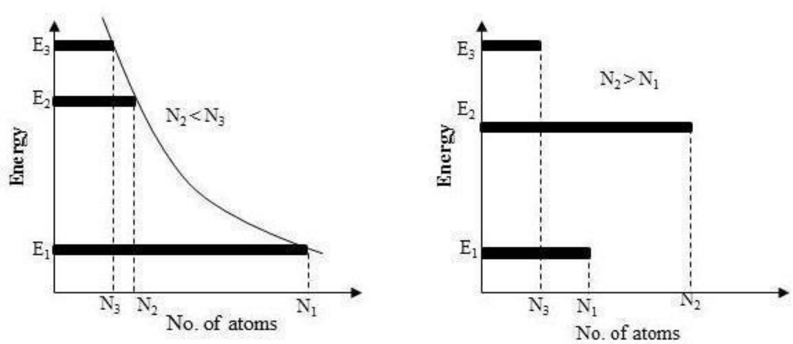
Population inversion:

When a system is in thermal equilibrium, the distribution of atoms in energy states at a given temperature follows the Boltzmann's law as

$$\frac{N_1}{N_2} = e^{\frac{hv}{KT}}$$

From the above equation, it is clear that the population is maximum in ground state as compared with excited state. i.e $N_1 > N_2$.

If the situation is reverse, i.e $N_2 > N_1$, there are more atoms in an excited state than the ground state as shown in fig(b), this condition is called **"Population inversion".**



When a suitable energy is supplied to the system, atoms get excited into E_3. After their lifetime 10^{-8} *seconds*, the atoms are transit to E_2. Due to more lifetime of an atom in state E_2(10^{-3} *secconds*), the atomsstay for longer time than compare with the state E_3. Due to the accumulation of atomsin E_2, the population inversion is established in between the E_2 and E_1 states.

Pumping methods :

The process of achieving population inversion is called pumping. Pumping can be classified into the following types based on the type of source of pumping.

1. **Optical pumping**: Here the atoms are excited with the help of photons emitted by an optical source. The atoms absorb energy from the photons and raises to excited state. Optical pumping is used in solid laser.
   Examples **:** Ruby Laser, Nd-YAG Laser

2. **Electrical pumping**: Electrical discharge pumping is used in gas lasers. Since gas lasers have very narrow absorption. The electrons are accelerated to very high velocities by strong electric field and they collide with gas atoms and these atoms are raised to excited state.
   Examples : Argon Laser, CO_2 Laser, He-Ne Laser

3. **Chemical pumping**: Due to some chemical reactions, the atoms may be raised to excited state. Examples : Dye Laser.

Metastable state :

It is an excited state of an atom with a longer life time than the other excited states. Atoms in the metastable state remain excited for a considerable time in the order of 10^{-6} to 10^{-3} seconds. Such relatively long-lived states are called as Metastable state. An atom can exist in a metastable energy level for a longer time before radiating than itcan in an ordinary energy level.

An atom can be excited to a higher level by supplying energy to it. Normally, excited atoms have short life times and release their energy in a matter of 10^{-8} seconds through spontaneous emission. It means atoms do not stay long to be stimulated. As a result, they undergo spontaneous emission and rapidly return to the ground level; thereby population inversion could not be established. In order to do so, the excited atoms are required to ‘wait’ at the upper energy level till a large number of atoms accumulate at that level, that is A Meta stable state.

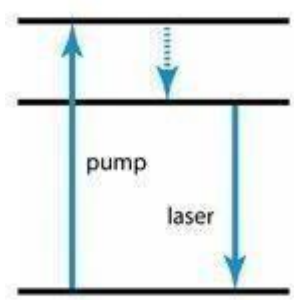
These levels lie in the forbidden gap of the host crystal. There could be no population inversion and hence no laser action, if metastable states don’t exist.

Optical Pumping: Three- and Four-Level Systems

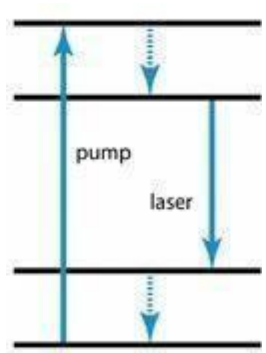
In a simple two-level system, it is not possible to obtain a population inversion with optical pumping because the system can absorb light (i.e., gain energy) only as long as population inversion, and thus light amplification, is not achieved. Essentially, the problem isstimulated emission caused by the pump light itself.

Inversion by optical pumping becomes possible when using a three-level system. Pump light with a shorter wavelength (higher photon energy) can transfer atoms from the ground state to the highest level. From there, spontaneous emission or a non-radiative process (e.g., involving phonons in a laser crystal) transfers atoms to an intermediate level, called the upper laser level. From that level down to the ground state, the laser transition with stimulated emission can occur. With sufficiently high pump intensity, population inversion for the laser transition can be reached as stimulated emission by the pump radiation is prevented by the transfer to the intermediate level.

Laser gain with a much lower excitation level is possible in a four-level system, such as Nd:YAG. Here, the lower level of the laser transition is somewhat above the ground state, anda rapid (most often non-radiative) transfer from there to the ground state keeps the population of the lower laser level very small. Therefore, a moderate population in the third level (the upper laser level), as achieved with a moderate pump intensity, is sufficient for laser amplification.



Three level



four level laser

Components of Lasers

Active Medium:

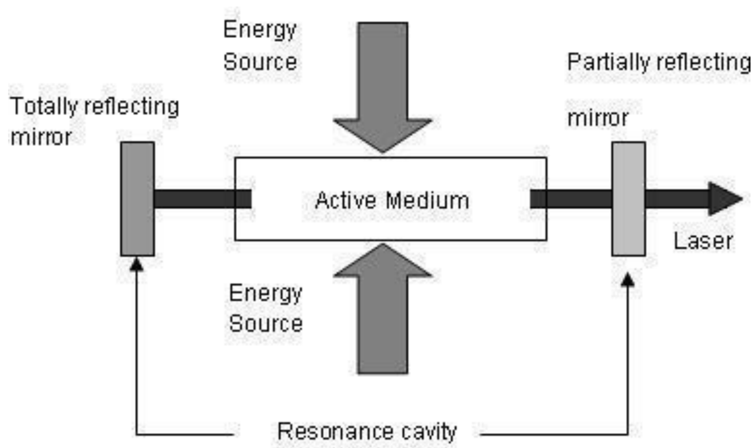
It is the material in which the laser action takes place. The active medium may be solid crystals such as liquid dyes, gases like CO_2 or Helium-Neon, or semiconductors such as GaAs. This medium decides the wavelength of laser radiation. Active mediums contain atoms which can produce more stimulated emission than spontaneous emission and cause amplification they are called “Active Centers”.

Pumping Energy Source (Excitation Mechanism):

Energy Source (Excitation mechanisms) pumps the active centers from ground state to excitedstate to achieve population inversion. The pumping by energy source can be optical, electricalor chemical depending on the active medium.

Resonance Cavity:

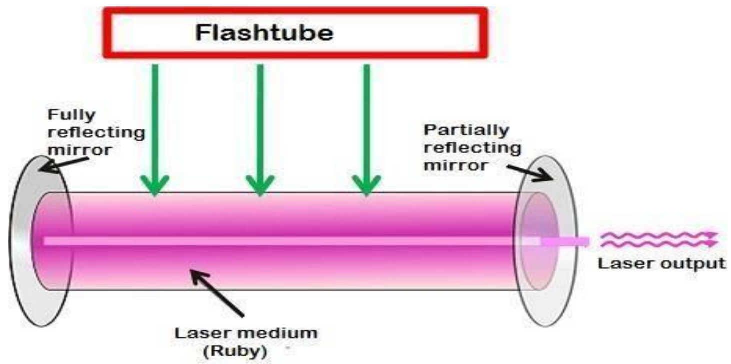
The optical resonator contains a pair of reflecting surfaces of which one is fully reflecting and the other partially reflecting. The active material is kept in between the two reflecting surfaces. Photons (light) emitted due to transitions between the energy states of the active material are bounced back and forth between the two reflecting surfaces, so the intensity of the light is increased. Finally the intense amplified beam called laser is coming out through the partial mirror as shown in the diagram.



Types of Lasers

1. Ruby Laser

The first working laser was built in 1960 by Maiman, using a ruby crytal and so called the Ruby laser. Ruby belongs to the family of gems consisting of Al_2O_3 with various types of impurities. For example, pink Ruby contains 0.05% Cr atoms. The schematic diagram of Ruby laser can be drawn as:



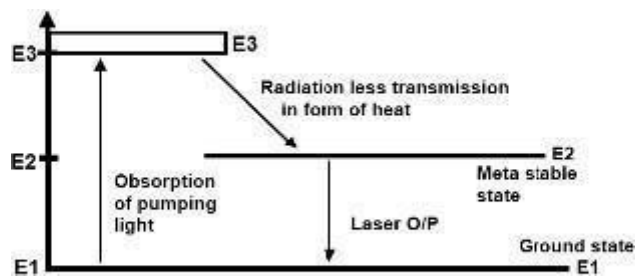
<u>Construction of Ruby Laser</u>

The ruby laser consists of a ruby rod. which is made of chromium doped ruby material. At the opposite ends of this rod there are two silver polished mirrors. Whose one is fully polished andother is partially polished. A spring is attached to the rod with fully polished end for adjustment of wave length of the laser light. Around the ruby rod a flash light is kept for the pump input. The whole assembly is kept in the glass tube. Around the neck of the glass tube the R.F source and switching control is designed in order to switch on and off the flash light for desired intervals.

<u>Operation of Ruby Laser:</u>

When we switch on the circuit the R.F operates. As a result, the flash of light is obtained aroundthe ruby rod this flash causes the electrons within ruby rod to move from lower energy band towards higher. This flash causes the electrons within ruby rod to move from lower energy band towards higher energy band. The population inversion take place at high energy band and electrons starts back to travel towards the lower energy band. During this movement the electron emits the laser light. This emitted light travels between the two mirrors where cross reflection takes place of this light. The stimulated laser light now escapes from partially polished mirror in shape of laser beam. The switching control of the R.F source is used to switch on and off the flash light so that excessive heat should not be generated due to very high frequency of the movement of the electron.

Energy Level Diagram for Ruby Laser



The above three level energy diagram show that in ruby lasers the absorption occurs, this makes raise the electrons from ground state E_1 to the band of level E_3 higher than E_1. At E_3 these excited levels are highly unstable and so the electrons decays rapidly to the level of E_2. This transition occurs with energy difference (E_1 – E_2) given up as heat (radiation less transmission). The level E_2 is very important for stimulated emission process and is known as Meta stable state. Electrons in this level have an average life time of about 5ms before they fall to ground state. After this the population inversion can be established between E_2 and E_1. The population inversion is obtained by opticalpumping of the ruby rod with a flash lamp. When the flash lamp intensity becomes large enough to create population inversion, then stimulated emission from the Meta stable level to the ground level occurs which result in the laser output.

Advantages of Ruby Lasers

- Beam diameter of the ruby laser is comparatively less than CO_2 gas lasers.
- Output power of Ruby laser is not as less as in He-Ne gas lasers.
- Since the ruby is in solid form therefore there is no chance of wasting material of

active medium.

- ❖ Construction and function of ruby laser is self-explanatory.

Disadvantages of Ruby Laser

- ❖ In ruby lasers no significant stimulated emission occurs, until at least half of the groundstate
- ❖ electrons have been excited to the Meta stable state.
- ❖ Efficiency of ruby laser is comparatively low.
- ❖ Optical cavity of ruby laser is short as compared to other lasers, which may beconsidered a disadvantage.

Applications of Ruby Laser

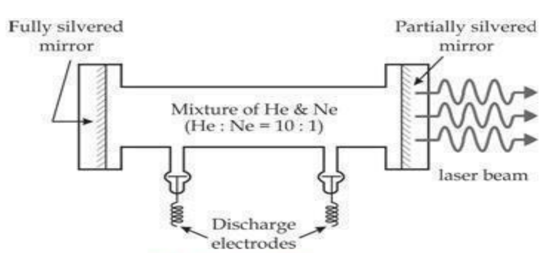
- ❖ Due to low output power they are class-I lasers and so may used as toys for children's.
- ❖ It can be used in schools, colleges, universities for science programs.
- ❖ It can be used as decoration piece & artistic display.

2. Helium -Neon (He-Ne) Laser

Construction:

Active medium:

It is a gas laser, which consists of a narrow quartz tube filled with a mixture of Helium and Neon gases in the ratio 10:1 respectively, at low pressure (~0.1 mm of Hg). Ne atoms act as active centers and responsible for the laser action, while He atoms are used to help in the excitation process. The length of the quartz tube is about 50 cm and the diameter is about 1 cm.



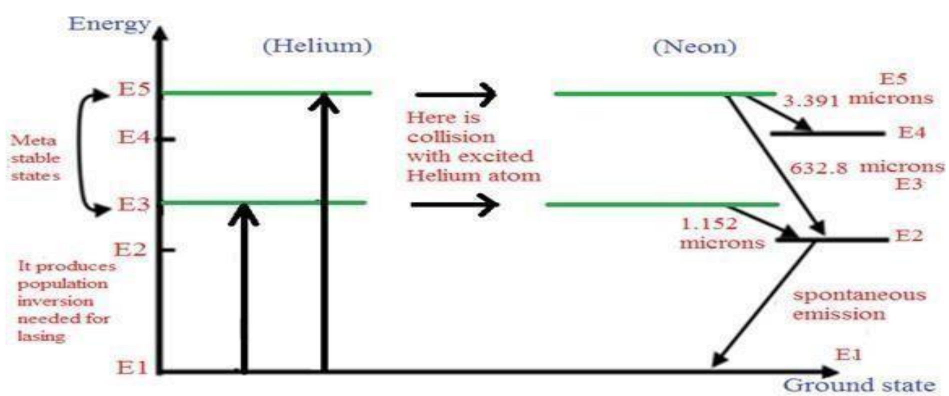
He-Ne LASER

Optical resonator:

To construct the optical resonator cavity, two parallel mirrors are placed at the ends of the quartz tube one of them is partly transparent while the other is fully reflecting. The spacing between the mirrors is adjusted such that it should be equal to the integral multiple of half- wavelengths of the laser light.

Pumping system:

The pumping is done through electrical discharge by using electrodes that are connected to a high frequency alternating current source.



The common helium-neon gas laser achieves a population inversion in a different way. A mixture of about 10 parts of Helium and 1 part of Neon at a low pressure is placed in a glass tube that has parallel mirrors, one of them partly transparent, at both ends. The spacing of the mirrors is equal to an integral number of half-wavelengths of the Laser light. An electric discharge is produced in the gas by means of electrodes outside the tube connected to a source of high-frequency alternating current, and collisions with electrons from the discharge excite He and Ne atoms to metastable states respectively above their ground states. Some of the excited He atoms transfer their energy to ground-state Ne atoms in collisions. The purpose of the He atoms is thus to help achieve a population inversionin the Ne atoms.

The laser transition in Ne is from the metastable state to an excited state with the emission of a 650nm of photon. Then another photon is spontaneously emitted in a transition to a lower metastable state; this transition yields only incoherent light.

Characteristics of He-Ne Laser

The He-Ne laser is a relatively low power device with an output in the visible red portion of the spectrum. The most common wavelength produced by He-Ne lasers is 632.8nm. Majority of He-Ne lasers generate less than 10m watt of power, but some can be obtained commercially with up to 50m watts of power. For He-Ne lasers the typical laser tube is from 10 to 100 cm in length and the life time of such a tube can be as high as 20,000 hours.

Applications of He-Ne Laser

The Helium-Neon gas laser is one of the most commonly used Laser today because of the following applications.

- Many schools / colleges / universities use this type of laser in their science programs and experiments.
- He-Ne lasers also used in super market checkout counters to read bar codes and QRcodes.
- The He-Ne lasers also used by newspapers for reproducing transmitted photographs.
- He-Ne lasers can be use as an alignment tool.
- It is also used in Guns for targeting.

Advantages of He-Ne Laser

- He-Ne laser has very good coherence property.
- He-Ne laser tube has very small length approximately from 10 to 100cm and best lifetime of 20.000 hours.
- Cost of He-Ne laser is less from most of other lasers.
- Construction of He-Ne laser is also not very complex.
- He-Ne laser provide inherent safety due to low power output.

Disadvantages of He-Ne Laser

- It is relatively low power device means its output power is low.
- He-Ne laser is low gain device.
- High voltage requirement can be considered its disadvantage.
- Escaping of gas from laser plasma tube is also its disadvantage.

Applications of Lasers

LASER Cutting

Laser is used as a tool to cut thin metal sheets by properly focusing the laser onto any particular area to be cut, for a longer time. Thus due to thermal effect the sheet is cut.

LASER Welding

In ordinary welding process the heat will be made to fall on the area to be welded, so that the material in that area will go to molten state. This on cooling will join the material. In this process the heat will spread all over the surroundings and will affect the other area of the material and hence the material gets damaged. This damage can be avoided by using laser welding. In laser welding the beam is focused onto the area to be welded and other areas remain unaffected. Without affecting the material the area to be welded alone melted and joined.

LASERS in Industry

Using high power lasers we can weld or melt any material. We can produce very small holes that cannot be done by mechanical drilling. Lasers can be used for cutting and for testing the quality of the materials. During laser welding and drilling there is no damage the structure of the materials. Lasers can be used for surface hardening techniques.

MEDICAL APPLICATIONS

Laser cosmetics surgery is used for removing tattoos, scars, stretch marks, sunspots, wrinkles and hairs.

1. Laser types used in dermatology:

   It include Ruby (694nm), pulsed diode arrays (810nm), Nd: YAG (1064nm) and Er : YAG (2940nm)

2. Laser eye surgery:

   Laser eye surgery is a medical procedure that uses a laser to reshape a surface of the eyes. This is done to correct short sightedness, long sightedness and astigmatism (uneven curvature of the eye surface).

3. Soft- Tissue surgery:

   a. In soft tissue laser surgery, a highly focused laser beam vapourises the soft tissue with the high water content.

b. Soft tissue laser surgery is used in a variety of applications which include general surgery, neuro surgery, ENT, dentistry and oral surgery.
c. Soft tissue laser surgery is also used in veterinary surgical fields.

4. Laser light therapy

   Laser light therapy involves exposure to laser light of specific variant. The light is administered for a prescribed amount of light. This is commonly used for skin diseases and disorders.

5. Laser is widely used for no-touch removal of tumors, especially of the brain and spinal cord
6. In dentistry, laser is used for tooth whitening and carries removal.

LASER SURGERY

A type of surgery that uses the cutting power of a laser beam to make bloodless cuts in tissue or remove a surface lens such as a skin tumor. There are a number of different types of lasers that differ in emitted light wavelengths and power ranges and in their ability to clot, cut or vapourise tissue. Among the commonly used lasers are pulsed dye laser, the YAG laser, the CO_2 laser and the argon laser.

Questions

1. Distinguish between Ordinary light and Laser light.
2. Distinguish between spontaneous and stimulated emission?
3. List out the characteristics of LASER.
4. What is meant by Spontaneous emission?
5. What is meant by stimulated emission?
6. What is meant by population inversion?
7. What is meant by pumping?
8. What are different methods of pumping?
9. What are the conditions required for laser action?
10. What are Einstein‘s coeffecients?
11. Define active medium.
12. What is meant by Optical resonator or Resonance cavity?
13. Explain construction and working of He-Ne Laser.
14. Explain construction and working of Ruby Laser.
15. Derive Einstein’s relation for stimulated emission and hence explain the existence of stimulated emission.
16. Discuss the applications of laser in various fields.